

Upper Tolerance Limits for Radiological Decision Making

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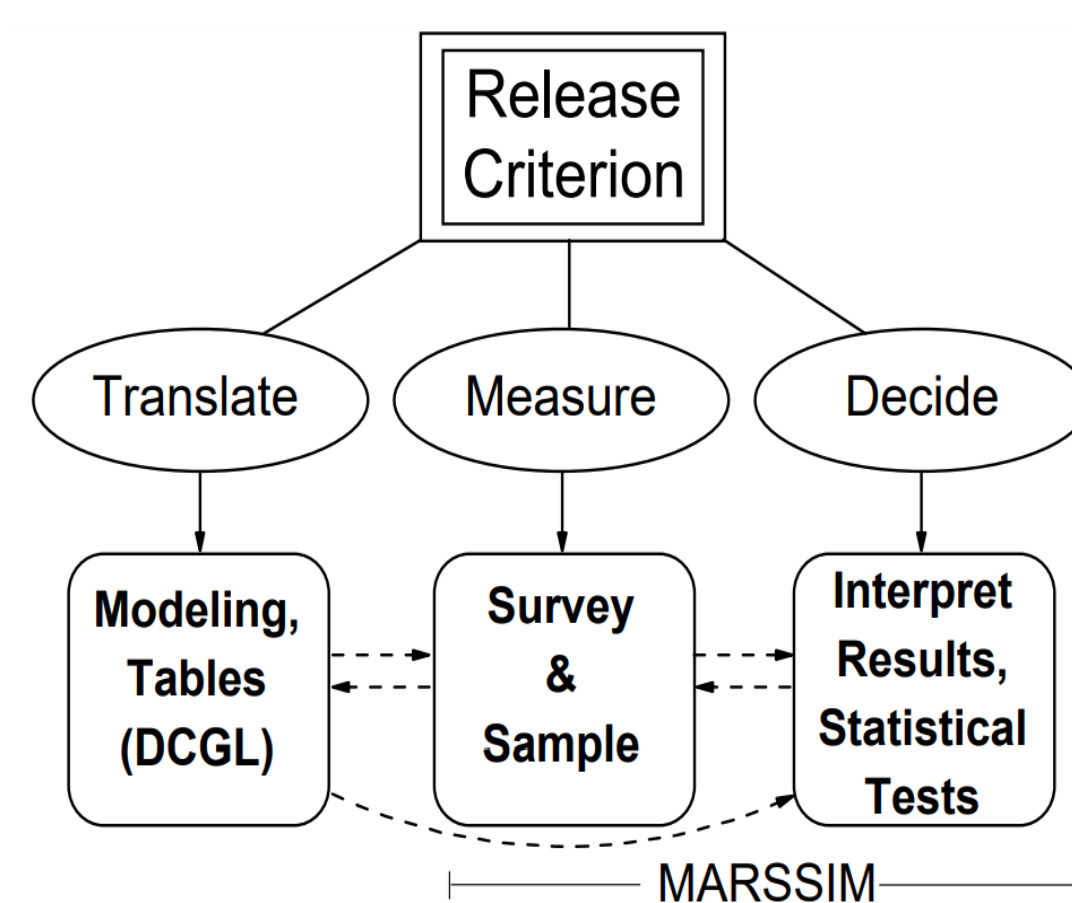
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Objective

Calculation and the use of upper tolerance limits (UTL) to quantify uncertainty for radiological decision making.

MARSSIM Framework

- The Multi-Agency Radiation Survey and Site Investigation Manual (MARSSIM) is a consensus document developed collaboratively by four Federal agencies with authority and control over radioactive materials: DOD, DOE, EPA, and NRC.
- MARSSIM provides standardized and consistent approaches for planning, conducting, evaluating, and documenting environmental radiological surveys.
- Making decisions based on sound statistical methods is one of the key three pillars of the release criterion, and quantification of uncertainty plays a key role in this.



Confidence Intervals (CI)

- Estimation of population parameters such as the population mean (μ) or standard deviation (σ).
- For example, sample mean (\bar{x}) is used to estimate μ .
- Confidence intervals quantify the uncertainty in estimating μ and provide plausible values of μ based on \bar{x} with some level of confidence ($1-\alpha$).

$$CI = \bar{x} \pm z \frac{\sigma}{\sqrt{n}}$$

- z is the confident coefficient from standard normal distribution based on the type 1 error rate α , and n is the sample size.
- This is a parametric approach where we assume the data follow a normal distribution.
- Non-parametric approaches like the bootstrap methods are also available for the calculation of confidence intervals for any population parameter.

Upper Tolerance Limits (UTL)

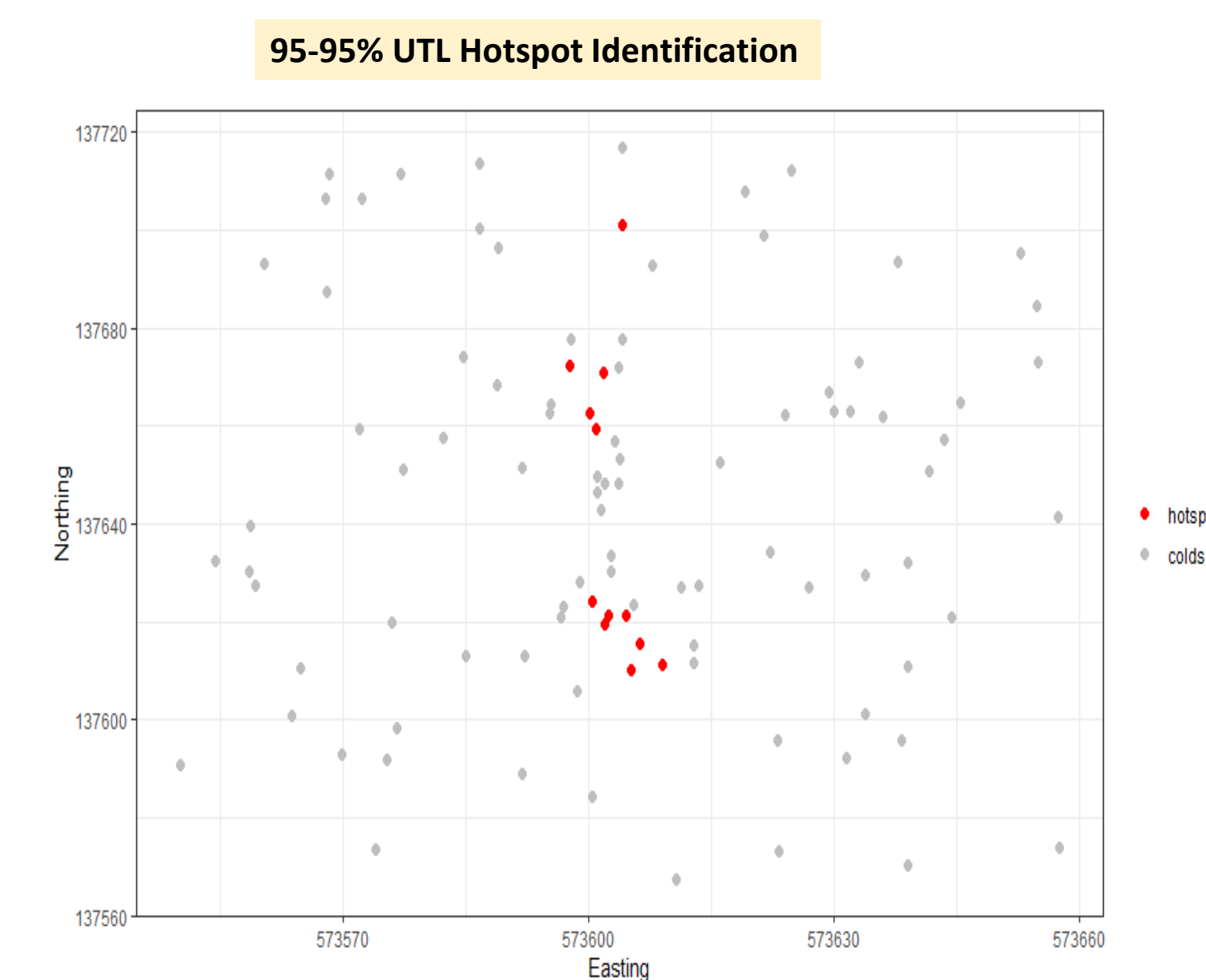
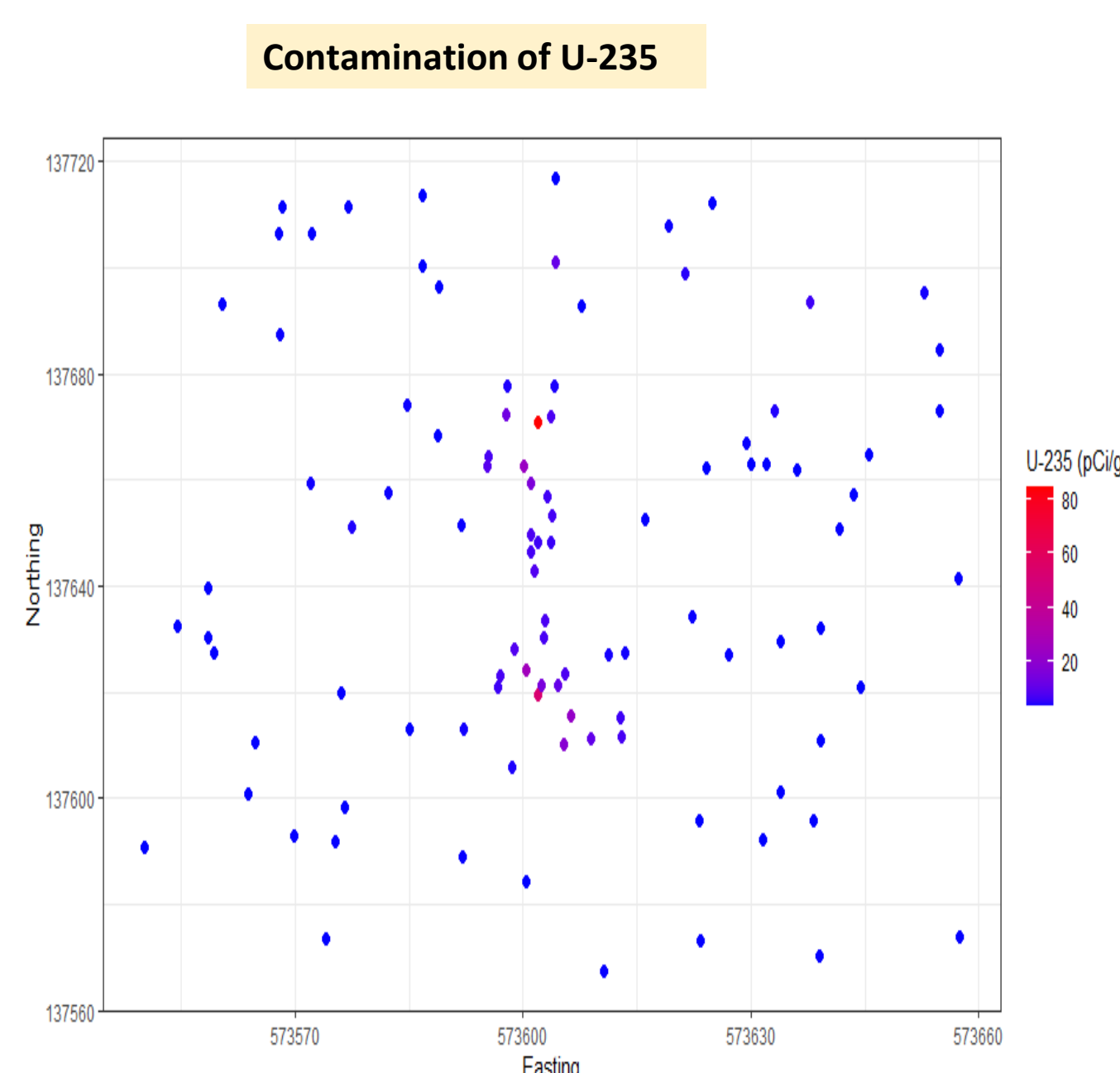
- There are situations where decisions should be based on comparing the upper tail of a distribution, rather than a mean or a parameter, to an action limit.
- Tolerance intervals also provide a way of quantifying the uncertainty in estimating the tails or proportion (percentile) of a population.
- Tolerance intervals are simply confidence intervals for a proportion or percentile of a population.
- For comparing upper tail of a distribution to an action limit, we use the one-sided UTL.
- A one-sided $UTL_{p,\alpha}$ is the value computed such that at least 100 p percentile of the population values is less than $UTL_{p,\alpha}$ with $100 \times (1-\alpha)$ confidence.
- $UTL_{p,\alpha}$ is simply a $1-\alpha$ upper confidence limit for the 100 p percentile of the population.
- Normal distribution:

$$UTL_{p,\alpha} = \bar{x} + t_{\{n-1, z_p \sqrt{n}, 1-\alpha\}} \frac{s}{\sqrt{n}}$$

- s is the sample standard and $t_{\{n-1, z_p \sqrt{n}, 1-\alpha\}}$ is p th percentile from of a non-central t distribution with $n-1$ df and non-central parameter $z_p \sqrt{n}$.
- Calculations for other distributions and exact non-parametric option for any continuous distribution are available.

Applications

- Site decommissioning (MARSSIM Class III area).
- Determining whether an area is contaminated (NUREG 1575).
- Comparison of mean and upper percentile to derived concentrations for uniform and non-uniform concentrations.
- Hotspot identification if there are no prior information.



Area of Future Research

- UTL for spatially dependent data.
- Calculation of mean and standard error after accounting for spatial autocorrelation.

$$UTL_{p,\alpha} = \bar{x}^* + t_{\{n^*-1, z_p \sqrt{n^*}, 1-\alpha\}} \frac{s^*}{\sqrt{n^*}}$$

- Calculation of effective sample size due pseudoreplication.