

# Grid Expansion Optimal Planning Tools (GridEO)

Manual for Users and Developers

December 2024

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## Abstract

This report describes Grid Expansion Optimal Planning Tools (GridEO). GridEO is a Python package for electric power grid capacity expansion modeling. With GridEO, the user can build and solve instances of the Capacity Expansion Planning (CEP) problem to determine an optimal plan of investment in capacity of various types of power grid equipment.

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## 1 Introduction

This report describes Grid Expansion Optimal Planning Tools (GridEO), a Python package designed for electric power grid capacity expansion modeling. GridEO enables users to build and solve instances of the Capacity Expansion Planning (CEP) problem, determining the optimal investment in generation, transmission, and storage resources. The tool co-optimizes generation and transmission investments, finding the minimum investment cost over long-term planning horizons.

GridEO is designed to account for various types of policy constraints, transmission limitations, and generation constraints, providing flexibility to model different system topologies with adjustable levels of granularity. It supports the inclusion of regulatory and environmental policies while ensuring optimal grid expansion planning that aligns with system reliability and decarbonization goals. The package enables detailed analyses by selecting the appropriate granularity for the power system topology, allowing planners to balance computational complexity with model accuracy.

## 2 Installation and Quick Start

GridEO is a Python package that can be installed from the source code with the typical Python package installation process. The code is stored in a repository at <https://gitlab.pnnl.gov/eset/LDES-plan/opt-modeling-experiments>, and the README file in that repository contains installation instructions. For reference we include the installation instructions here.

First, place the code into your file system, either by downloading or cloning, into a directory `<install_dir>`. e.g., clone the repository:

```
$ cd <install_dir>
$ git clone https://gitlab.pnnl.gov/eset/LDES-plan/opt-modeling-experiments.git
```

Next, install the GridEO Python package from the code:

```
$ cd <install_dir>/opt-modeling-experiments
$ pip install -e .
```

Next, run an example script showing how to import GridEO Python objects:

```
$ python <install_dir>/opt-modeling-experiments/examples/import.py
```

To run the main script, do:

```
$ python input_output_examples/pyomo_read_wecc_data.py
```

This may fail, without further arguments, so to get the full syntax, do:

```
$ python input_output_examples/pyomo_read_wecc_data.py --help
```

An important argument for input data is `--in_dir <IN_DIR>`. This argument specifies a directory containing a database. An example database is in a repository at [https://tanuki.pnnl.gov/Long-Term-Storage-Planning-Models/capacity\\_expansion\\_planning\\_database.git](https://tanuki.pnnl.gov/Long-Term-Storage-Planning-Models/capacity_expansion_planning_database.git). The database repository can be cloned by:

```
$ cd <IN_DIR>
$ git clone https://tanuki.pnnl.gov/Long-Term-Storage-Planning-Models/
capacity_expansion_planning_database.git
```

The output directory can also be specified, with the `--out_dir <OUT_DIR>`. You can then run the main script, with input from the specified database and directing output to the specified directory:

```
$ python input_output_examples/pyomo_read_wecc_data.py
--in_dir <IN_DIR> --out_dir <OUT_DIR>
```

## 3 Nomenclature

### Units

\$ US dollars, money

h hours, time

y years, time

rad radians, angle

MWh megawatt-hours, electrical energy

MW megawatts, power

BTU British thermal units, fuel energy content

TCO<sub>2e</sub> tons CO<sub>2</sub> equivalent, greenhouse gas

**Symbol convention** Each symbol is a main letter or word with subscripts and superscripts. Upper case letters are used for sets, input data parameters, and functions. Lower case letters are used for set elements and variables. Greek letters are used where traditional, such as  $\theta$  for voltage angle. Subscripts are used for indices. Superscripts further define the symbols. Sets are represented by an upper case main letter, subsets of the same index set use the same main letter, and indices and distinguished elements of a set use lower case letters of the same letter. Each main letter is associated with a certain type of quantity and units, e.g.,  $P$  is for power (MW),  $C$  is for power capacity (MW),  $E$  is for energy (MWh).

**Main letter convention**

$e, E$  Energy (MWh)

$p, P$  Power generation consumption or transformation (MW)

$c, C$  Power capacity (MW)

$D$  Time duration (h, y)

$F$  Cost, penalty, or value coefficient (\$ costs over various units)

$g, G$  Carbon emission rate (TCO<sub>2e</sub>/y)

$A$  Miscellaneous parameters (various units, including unitless)

$i, I$  Projects

$t, T$  Operational scale time intervals, time points (e.g., hours)

$y, Y$  Investment scale time periods (e.g., years, 5 year periods, decades)

$n, N$  Zones

$h, H$  Horizons

$\theta$  Voltage angle

$s, S$  Stochastic scenarios

$w$  Cost or penalty incurred or value realized (\$)

$z, Z$  Number of units

### **Superscript convention**

*ld* Load

*et* Energy target

*ct* Carbon tax

*pr* Planning reserve

*cc* Carbon cap

*tr* Transmission

*st* Storage

*ui* Unitized investment

*uo* Unitized operation

*r* Resistance

*x* Reactance

*g* Conductance

*b* Susceptance

*bdc* Susceptance in a DC model (also called admittance)

*w* Weight

*hydro* Hydropower

*eff* Efficiency

*ch* Charge

*dch* Discharge

*gci* Generation carbon intensity

*nqc* Net qualifying capacity

*sch* Scheduled, or specified

*efc* Effective

*en* Energy

*p* Power

*avail* Available

*sch* Exogenous schedule of capacity



*min* Minimum value  
*max* Maximum value  
*inc* Capacity increase from previous period  
*dec* Capacity decrease from previous period  
*ret* Endogenous retirement of existing capacity  
*inv* Endogenous investment in new capacity  
*acc* Accumulated operable capacity  
*ext* Existing capacity  
*new* New capacity  
*ante* Anterior - before  
*post* Posterior - after  
*start* Start  
*end* End  
*inv* Investment  
*aic* Annualized investment cost  
*df* Discount factor  
*ret* Retirement  
*pc* Power cost  
*fp* Fuel price  
*hr* Heat rate  
*vom* Variable operations and maintenance cost  
*py* Power imbalance penalty  
+ Positive part, or violation of an upper bound  
– Negative part, or violation of a lower bound

### Sets and indices

$y \in Y$  Investment time periods, e.g., years, decades.

$h \in H$  Operational time horizons, e.g., days, weeks.

$t \in T$  Operational time intervals, e.g., hours. Each interval is unique in the whole model horizon and occurs in one specific investment period.

$n \in N$  Zones for power balance and operational and investment policy constraints, i.e., collections of non-transmission projects, may or may not be geographically defined.

$i \in I$  Projects, including generation projects and transmission projects, generally characterized by a specific technology in a specific load zone or spanning a pair of load zones.

$s \in S$  Stochastic scenarios.

### Subsets

$H_i \subset H$  Horizons associated with project  $i$

$H^{et} \subset H$  Energy target horizons

$H_n^{et} \subset H$  Energy target horizons for energy target zone  $n$ .

$I^{gn} \subset I$  Generation projects

$I^{st} \subset I$  Storage projects

$I^{tr} \subset I$  Transmission projects

$I^{new} \subset I$  Projects representing new capacity

$I^{ext} \subset I$  Projects representing existing capacity

$I^{ui} \subset I$  Projects with unitized investment

$I^{uo} \subset I$  Projects with unitized operation

$I^{cgnl}$  Projects with capacity type 'gen new lin'

$I^{cgs}$  Generation projects with capacity type 'gen spec'

$I^{cgrl}$  Generation projects with capacity type 'gen ret lin'

$I^{csnl}$  Generation projects with capacity type 'stor new lin'

$I^{css}$  Generation projects with capacity type 'stor spec'

$I^{ogs}$  Generation projects with operational type 'gen simple'

$I^{os}$  Generation projects with operational type 'storage'

$I^{ogv} \subset I$  Generation projects with operational type 'gen var'

$I^{ogh} \subset I$  Generation projects with operational type 'gen hydro'

$I_n \subset I$  Non-transmission projects in zone  $n$

$I_n^{fr} \subset I$  Transmission projects directed from zone  $n$

$I_n^{to} \subset I$  Transmission projects directed to zone  $n$

$N_i \subset N$  Zones containing non-transmission project  $i$

$N^{ld} \subset N$  Load zones

$N^{pr} \subset N$  Planning reserve zones

$N^{cc} \subset N$  Carbon cap zones

$N^{ct} \subset N$  Carbon tax zones

$N^{et} \subset N$  Energy target zones

$Y_{iy}^{ante} \subset Y$  Investment periods  $y'$  before period  $y$  such that investment in capacity in project  $i$  in period  $y'$  is operable in period  $y$

$Y_{iy}^{post} \subset Y$  Investment periods  $y'$  after period  $y$  such that investment in capacity in project  $i$  in period  $y$  is operable in period  $y'$

### Special set elements

$n_n^{etld} \in N^{ld}$  The load zone associated with energy target zone  $n$

$n_i^{ld} \in N$  Load zone of generation project  $i$  - each project may be in multiple zones but should be in exactly one load zone

$n_i^{fr} \in N$  From (i.e., origin, sending, etc.) zone of transmission project  $i$

$n_i^{to} \in N$  To (i.e., destination, receiving, etc.) zone of transmission project  $i$

$t_{ht}^{prev} \in T$  Previous time point to time  $t$  in horizon  $h$ , i.e., immediately preceding

$t_{ht}^{next} \in T$  Next time point to time  $t$  in horizon  $h$ , i.e., immediately following

$y^{start} \in Y$  The starting investment period in the model horizon

$y_t \in Y$  The investment period containing operational interval  $t$

$y_h \in Y$  The investment period containing horizon  $h$

## Parameters

- $A_i^x$  Impedance of transmission project  $i$
- $A_i^r$  Resistance of transmission project  $i$
- $A_i^g$  Conductance of transmission project  $i$
- $A_i^b$  Susceptance of transmission project  $i$
- $A_i^{bdc}$  Susceptance of transmission project  $i$  for the DC power flow model, expressed as power flow induced by 1 rad angle difference (MW)
- $A_y^w$  Weight of investment period  $y$  (unitless)
- $A_t^w$  Weight of operational interval  $t$  (unitless)
- $A_y^{df}$  Discount factor in year  $y$  (unitless)
- $A_{it}^{fp}$  Fuel price for generation project  $i$  in interval  $t$  (\$/BTU)
- $A_i^{hr}$  Heat rate for generation project  $i$  (BTU/MWh)
- $A_{it}^{cf}$  Normalized hourly output of each  $g \in I^{ogv}, I^{ogh}$  at each time  $t$  (unitless)
- $A_{it}^{avail}$  Availability of project  $i$  in timepoint  $t$  (unitless)
- $A_{it}^{efc}$  Effective capacity factor of project  $i$  in interval  $t$  (unitless)
- $A_{ih}^{hydro,max}$  Hydro maximum average capacity factor for horizon  $h$  (unitless)
- $A_{it}^{hydro,max}$  Hydro maximum capacity factor for interval  $t$  (unitless)
- $A_{it}^{hydro,min}$  Hydro minimum capacity factor for interval  $t$  (unitless)
- $A_i^{eff,ch}$  Charging efficiency of storage project  $i$  (unitless)
- $A_i^{eff,dch}$  Discharging efficiency of storage project  $i$  (unitless)
- $A_i^{gci}$  Generation carbon intensity of project  $i$  (tCO<sub>2</sub>e/MWh)
- $A_{nh}^{et}$  Energy target for energy target zone  $n$  in energy target horizon  $h$ , expressed as a fraction of the load in the associated load zone over the horizon  $h$  (unitless)
- $A_i^{nqc}$  project  $i$  net qualifying capacity factor (unitless)
- $C_{iy}^{sch}$  Specified (scheduled) capacity for project  $i$  in year  $y$
- $C_{iy}^{inv,min}$  Minimum incremental capacity investment of project  $i$  in year  $y$ . (MW)

$C_{iy}^{inv,max}$  Maximum incremental capacity investment of project  $i$  in year  $y$ . (MW)

$C_{iy}^{acc,max}$  Maximum accumulated capacity investment of project  $j$ . (MW)

$C_{iy}^{acc,min}$  Minimum accumulated capacity investment of project  $j$ . (MW)

$C_i^{tr}$  Capacity of transmission project  $i$  (MW)

$C_{ny}^{pr}$  planning reserve requirement for planning reserve zone  $n$  in period  $y$  (MW)

$D_y$  Duration of investment period  $y$  (y)

$D_t$  Duration of operational interval  $t$  (h)

$D_y^{start}$  Start time of investment period  $y$  (y)

$D_y^{end}$  End time of investment period  $y$  (y)

$D_i^{life}$  Lifetime of project  $i$  (y)

$F_{iy}^{aic}$  Annualized investment costs for project  $i$  in year  $y$  (\$/MW)

$F_i^{vom}$  Variable operations and maintenance costs for generation project  $i$  (\$/MWh)

$F_{iy}^{aic,en}$  Annualized investment cost of energy capacity (\$/MWh)

$F_{iy}^{inv}$  Capacity investment cost for project  $i$  in period  $y$  (\$/MW)

$F_{iy}^{en,inv}$  Energy capacity investment cost for storage project  $i$  in period  $y$  (\$/MWh)

$F_{iy}^{ret}$  Capacity retirement cost for project  $i$  in period  $y$  (\$/MW)

$F_{iy}^{en,ret}$  Energy capacity retirement cost for storage project  $i$  in period  $y$  (\$/MW)

$F_{it}^{pc}$  Power cost for project  $i$  in operational interval  $t$  (\$/MWh)

$F_t^{p+}$  Over-generation penalty in operational interval  $t$  (\$/MWh)

$F_t^{p-}$  Under-generation penalty in operational interval  $t$  (\$/MWh)

$F_{ny}^{cc}$  Carbon cap violation penalty in zone  $n$  in period  $y$  (\$/tCO<sub>2</sub>e)

$F_n^{et}$  Energy target violation penalty in energy target zone  $n$  (\$/MWh)

$F_{ny}^{ctr}$  Carbon tax rate (\$/tCO<sub>2</sub>e)

$G_{ny}^{cc}$  Carbon cap of zone  $n$  in period  $y$  (tCO<sub>2</sub>e/h)

$P_t^{ld}$  System total load in time interval  $t$

$P_{nt}^{ld}$  load at load zone  $n$  timepoint  $t$

## Variables

- $c_{iy}^{acc}$  Accumulated capacity of project  $i$  operating in investment period  $y$  (MW)
- $c_{iy}^{inv}$  Investment in new capacity of project  $i$  in investment period  $y$  (MW)
- $c_{iy}^{ret}$  Retirement of existing capacity of project  $i$  in investment period  $y$  (MW)
- $c_{iy}^{inc}$  Increase in accumulated capacity of project  $i$  in investment period  $y$  (MW)
- $c_{iy}^{dec}$  Decrease in accumulated capacity of project  $i$  in investment period  $y$  (MW)
- $c_{iy}^{acc}$  Accumulated energy capacity of storage project  $i$  operating in investment period  $y$  (MW)
- $c_{iy}^{en,inv}$  Investment in new energy capacity of storage project  $i$  in investment period  $y$  (MW)
- $c_{iy}^{en,ret}$  Retirement of existing energy capacity of storage project  $i$  in investment period  $y$  (MW)
- $c_{iy}^{en,inc}$  Increase in accumulated energy capacity of storage project  $i$  in investment period  $y$  (MW)
- $c_{iy}^{en,dec}$  Decrease in accumulated energy capacity of storage project  $i$  in investment period  $y$  (MW)
- $e_{it}$  Stored energy, i.e., state of charge, of storage project  $i$  at the end of the interval  $t$  (MWh)
- $g_{ny}^{cc+}$  Carbon cap constraint violation in carbon cap zone  $n$  in period  $y$  (TCO<sub>2</sub>e/y)
- $p_{it}$  power output from generation project  $i$  in interval  $t$  or flow on transmission project  $i$  in interval  $t$  (MW)
- $p_{it}^{ch}$  Charging power of storage project  $i$  in interval  $t$  (MW)
- $p_{it}^{dch}$  Discharging power of storage project  $i$  in interval  $t$  (MW)
- $p_t^+$  excess system wide power supply in interval  $t$  (MW)
- $p_t^-$  excess system wide power demand in interval  $t$  (MW)
- $p_{nt}^+$  excess power supply in zone  $n$  in interval  $t$  (MW)
- $p_{nt}^-$  excess power demand in zone  $n$  in interval  $t$  (MW)
- $p_{nh}^{et+}$  Energy target constraint violation for energy target zone  $n$  in energy target horizon  $h$  (MW)
- $p_{ny}^{et+}$  Energy target constraint violation for energy target zone  $n$  in period  $y$  (MW)

- $w$  Total cost objective for minimization (\$)
- $w^{cc}$  Carbon cap constraint violation penalty incurred (\$)
- $w^{ct}$  Carbon tax incurred (\$)
- $w^{et}$  Energy target constraint violation penalty incurred (\$)
- $z_{iy}^{inv}$  Investment number of units of project  $i$  in period  $y$  (unitless)
- $z_{iy}^{ret}$  Retirement number of units of existing capacity of project  $i$  in investment period  $y$  (unitless)
- $z_{iy}^{acc}$  Accumulated number of units of project  $i$  operating in investment period  $y$  (unitless)
- $\theta_{nt}$  voltage angle in zone  $n$  in interval  $t$  (rad)

## 4 Construction of Model Data from Inputs

The sets  $Y_{iy}^{ante}$  and  $Y_{iy}^{post}$  of anterior and posterior investment periods are defined by the operating lifetime  $D_i^{life}$  of new capacity investment of project  $i$ . Specifically, we define the start time  $D_y^{start}$  and the end time  $D_y^{end}$  of investment period  $y$  in terms of the duration  $D_y$ :

$$D_y^{end} = \sum_{y' \leq y} D_{y'} \quad \forall y \in Y \quad (1)$$

$$D_y^{start} = D_{y-1}^{end} \quad \forall y \in Y, y > y^{start} \quad (2)$$

$$D_y^{start} = 0 \quad \forall y = y^{start} \quad (3)$$

Then the posterior investment periods  $Y_{iy}^{post}$  for capacity of project  $i$  built in period  $y$  are those later periods  $y'$  where the capacity remains operating, i.e., where the operating lifetime does not extend past the end of period  $y'$  if it is started at the start of period  $y$ :

$$Y_{iy}^{post} = \{y' \geq y : D_y^{start} + D_i^{life} \leq D_{y'}^{end}\} \quad \forall i \in I, y \in Y \quad (4)$$

And the anterior investment periods  $Y_{iy}^{ante}$  are those earlier periods  $y'$  where capacity built in period  $y'$  remains operating in period  $y$ :

$$Y_{iy}^{ante} = \{y' \leq y : D_{y'}^{start} + D_i^{life} \leq D_y^{end}\} \quad \forall i \in I, y \in Y \quad (5)$$

We note that the set maps  $Y^{ante}$  and  $Y^{post}$  are adjoints in a sense:

$$\{(y, y') : y' \in Y_{iy}^{ante}\} = \{(y, y') : y \in Y_{iy'}^{post}\} \quad \forall i \in I \quad (6)$$

The capacity investment cost  $F_{iy}^{inv}$  is determined by the annualized investment cost  $F_{iy}^{aic}$ , with investment period weighting  $A_{y'}^w$  and discount factor  $A_{y'}^{df}$  applied over all posterior investment periods  $y'$ :

$$F_{iy}^{inv} = \sum_{y' \in Y_{iy}^{post}} A_{y'}^{df} A_{y'}^w F_{iy}^{aic} \quad \forall i \in I^{new}, y \in Y \quad (7)$$

The power generation cost coefficient  $F_{it}^{pc}$  is determined by the fuel price  $F_{it}^{fp}$ , the heat rate  $A_i^{hr}$ , and the variable operations and maintenance cost  $A_i^{vom}$ , with investment period weighting  $A_y^w$ , discount factor  $A_y^{df}$ , and operating interval weighting  $A_t^w$  applied:

$$F_{it}^{pc} = A_y^{df} A_y^w A_t^w (F_{it}^{fp} A_i^{hr} + F_i^{vom}) \quad \forall i \in I, t \in T, y = y_t \quad (8)$$

The over- and under-generation penalty coefficients  $F_t^{p+}$  and  $F_t^{p-}$  are determined by scalar penalty coefficient  $F^{p+}$  and  $F^{p-}$ , with investment period weighting  $A_y^w$ , discount factor  $A_y^{df}$ , and operating interval weighting  $A_t^w$  applied:

$$F_t^{p+} = A_y^{df} A_y^w A_t^w F^{p+} \quad \forall t \in T, y = y_t \quad (9)$$

$$F_t^{p-} = A_y^{df} A_y^w A_t^w F^{p-} \quad \forall t \in T, y = y_t \quad (10)$$

The effective capacity factor  $A_{it}^{efc}$  is determined as a product of capacity factors representing weather conditions and outage.

The total load  $P_t^{ld}$  is computed from the zonal load  $P_{nt}^{ld}$  by summing over load zones  $n$ :

$$P_t^{ld} = \sum_{n \in N^{ld}} P_{nt}^{ld} \quad (11)$$

## 5 Model: Variables, Constraints, Objective

### 5.1 Objective

The objective  $w$  for minimization is the total of power and energy capacity investment and retirement costs, power generation costs, and penalties on system-wide and zonal over- and under-generation, as well as penalties on violations of carbon cap, carbon tax, and energy target constraints:

$$\begin{aligned} w = & \sum_{i \in I^{new}, y \in Y} F_{iy}^{inv} c_{iy}^{inv} + \sum_{i \in I^{ext}, y \in Y} F_{iy}^{ret} c_{iy}^{ret} \\ & + \sum_{i \in I^{st} \cap I^{new}, y \in Y} F_{iy}^{en,inv} c_{iy}^{en,inv} + \sum_{i \in I^{st} \cap I^{ext}, y \in Y} F_{iy}^{en,ret} c_{iy}^{en,ret} \\ & + \sum_{i \in I^{gn}, t \in T} F_{it}^{pc} p_{it} + \sum_{t \in T} F_t^{p+} p_t^+ + \sum_{t \in T} F_t^{p-} p_t^- \\ & + \sum_{n \in N^{ld}, t \in T} F_t^{p+} p_{nt}^+ + \sum_{n \in N^{ld}, t \in T} F_t^{p-} p_{nt}^- + w^{cc} + w^{ct} + w^{et} \quad (12) \end{aligned}$$



## 5.2 Power Capacity Tracking, Investment and Retirement

For technologies  $i \in I^{ext}$  representing existing capacity, we have an exogenous capacity schedule with endogenous retirements. The retirements are subject to bounds, and the accumulated capacity is determined by the scheduled capacity minus the retirements in earlier periods:

$$c_{iy}^{acc} = C_{iy}^{sch} - \sum_{y' \leq y} c_{iy'}^{ret} \quad i \in I^{ext}, y \in Y \quad (13)$$

$$C_{iy}^{ret,min} \leq c_{iy}^{ret} \leq C_{iy}^{ret,max} \quad \forall i \in I^{ext}, y \in Y \quad (14)$$

For technologies  $i \in I^{new}$  representing new capacity, we have endogenous investment, but retirement is determined by an exogenous lifetime. The capacity investment is subject to bounds, and the accumulated capacity is determined by the prior investments that are still within their lifetimes:

$$c_{iy}^{acc} = \sum_{y' \in Y_{iy}^{ante}} c_{iy'}^{inv} \quad i \in I^{new}, y \in Y \quad (15)$$

$$C_{iy}^{inv,min} \leq c_{iy}^{inv} \leq C_{iy}^{inv,max} \quad i \in I^{new}, y \in Y \quad (16)$$

For all technologies  $i \in I$ , we allow bounds on the accumulated capacity and on the capacity increase and decrease in each period:

$$c_{iy}^{acc} - c_{i,y-1}^{acc} = c_{iy}^{inc} - c_{iy}^{dec} \quad \forall i \in I, y \in Y \quad (17)$$

$$C_{iy}^{acc,min} \leq c_{iy}^{acc} \leq C_{iy}^{acc,max} \quad \forall i \in I, y \in Y \quad (18)$$

$$C_{iy}^{inc,min} \leq c_{iy}^{inc} \leq C_{iy}^{inc,max} \quad \forall i \in I, y \in Y \quad (19)$$

$$C_{iy}^{dec,min} \leq c_{iy}^{dec} \leq C_{iy}^{dec,max} \quad \forall i \in I, y \in Y \quad (20)$$

## 5.3 Energy Capacity Tracking, Investment and Retirement, for Storage Projects

Storage projects are characterized by energy capacity as well as power capacity, and therefore, a similar model of energy capacity investment, retirement,

accumulation, increase, and decrease is formulated

$$c_{iy}^{en,acc} = C_{iy}^{en,sch} - \sum_{y' \leq y} c_{iy'}^{en,ret} \quad \forall i \in I^{st} \cap I^{ext}, y \in Y \quad (21)$$

$$C_{iy}^{en,ret,min} \leq c_{iy}^{en,ret} \leq C_{iy}^{en,ret,max} \quad \forall i \in I^{st} \cap I^{ext}, y \in Y \quad (22)$$

$$c_{iy}^{en,acc} = \sum_{y' \in Y_{iy}^{ante}} c_{iy'}^{en,inv} \quad \forall i \in I^{st} \cap I^{new}, y \in Y \quad (23)$$

$$C_{iy}^{en,inv,min} \leq c_{iy}^{en,inv} \leq C_{iy}^{en,inv,max} \quad \forall i \in I^{st} \cap I^{new}, y \in Y \quad (24)$$

$$c_{iy}^{en,acc} - c_{i,y-1}^{en,acc} = c_{iy}^{en,inc} - c_{iy}^{en,dec} \quad \forall i \in I^{st}, y \in Y \quad (25)$$

$$C_{iy}^{en,acc,min} \leq c_{iy}^{en,acc} \leq C_{iy}^{en,acc,max} \quad \forall i \in I^{st}, y \in Y \quad (26)$$

$$C_{iy}^{en,inc,min} \leq c_{iy}^{en,inc} \leq C_{iy}^{en,inc,max} \quad \forall i \in I^{st}, y \in Y \quad (27)$$

$$C_{iy}^{en,dec,min} \leq c_{iy}^{en,dec} \leq C_{iy}^{en,dec,max} \quad \forall i \in I^{st}, y \in Y \quad (28)$$

Storage energy capacity is bounded by limits depending on the power capacity and prescribed minimum and maximum storage duration:

$$D_i^{st,min} c_{iy}^{en,acc} \leq c_{iy}^{en,acc} \leq D_i^{st,max} c_{iy}^{en,acc} \quad \forall i \in I^{st}, y \in Y \quad (29)$$

#### 5.4 Project Capacity Unit Modeling

Projects  $i \in I^{ui}$  have a concept of units, and investment and retirement in these projects can be expressed as a number of units:

$$c_{iy}^{inv} = C_i^u z_{iy}^{inv} \quad \forall i \in I^{new} \cap I^{ui} \quad (30)$$

$$c_{iy}^{ret} = C_i^u z_{iy}^{ret} \quad \forall i \in I^{ext} \cap I^{ui} \quad (31)$$

The number of units in operation can then be tracked for new and existing projects similar to the capacity:

$$z_{iy}^{acc} = Z_{iy}^{sch} - \sum_{y' \leq y} z_{iy'}^{ret} \quad \forall i \in I^{ext}, y \in Y \quad (32)$$

$$z_{iy}^{acc} = \sum_{y' \in Y_{iy}^{ante}} z_{iy'}^{inv} \quad \forall i \in I^{new}, y \in Y \quad (33)$$

The number of units is subject to bounds:

$$Z_{iy}^{inv,min} \leq z_{iy}^{inv} \leq Z_{iy}^{inv,max} \quad \forall i \in I^{new} \cap I^{ui} \quad (34)$$

$$Z_{iy}^{ret,min} \leq z_{iy}^{ret} \leq Z_{iy}^{ret,max} \quad \forall i \in I^{ret} \cap I^{ui} \quad (35)$$

The number of units can be required to be an integer:

$$z_{iy}^{inv} \in \{0, 1, \dots\} \quad \forall i \in I^{new} \cap I^{ui} \quad (36)$$

$$z_{iy}^{ret} \in \{0, 1, \dots\} \quad \forall i \in I^{ext} \cap I^{ui} \quad (37)$$

If equations (36) and (37) are used, then the resulting model is a mixed integer programming problem, while otherwise it remains a linear programming problem.

## 5.5 Generation Power Limits

Generator power output is constrained by bounds determined by the accumulated capacity, scaled by an effective availability factor:

$$0 \leq p_{it} \leq A_{it}^{efc} c_{iy}^{acc} \quad \forall i \in I^{gn}, y \in Y, t \in T_y \quad (38)$$

## 5.6 Storage Operations

Storage state of charge evolves from the end of one operational time interval to the next by the net inflow of energy resulting from charging and discharging:

$$e_{it} = e_{it'} + D_t \left( A_i^{eff,ch} p_{it}^{ch} - p_{it}^{dch} / A_i^{eff,dch} \right) \quad \forall i \in I^{st}, h \in H_i, t \in T_h, t' = t_{ht}^{prev} \quad (39)$$

Storage state of charge is subject to bounds determined by the energy capacity:

$$0 \leq e_{it} \leq c_{iy}^{en,acc} \quad \forall i \in I^{st}, y \in Y, t \in T \quad (40)$$

Storage charging and discharging power are subject to bounds determined by the capacity:

$$0 \leq p_{it}^{ch} \leq c_{iy}^{acc} \quad \forall i \in I^{st}, y \in Y, t \in T \quad (41)$$

$$0 \leq p_{it}^{dch} \leq c_{iy}^{acc} \quad \forall i \in I^{st}, y \in Y, t \in T \quad (42)$$

## 5.7 Hydro Operations

Hydro power is limited by bounds depending on the capacity

$$A_{it}^{hydro,min} c_{iy}^{acc} \leq p_{it} \leq A_{it}^{hydro,max} c_{iy}^{acc} \quad \forall i \in I, y \in Y, t \in T_y \quad (43)$$

Hydro power may be subject to total energy budget constraints, each of which applies to a given horizon:

$$\sum_{t \in T_h} D_t p_{it} \leq A_{ih}^{hydro,max} D_h c_{iy}^{acc} \quad \forall i \in I, h \in H_i, y = y_h \quad (44)$$

## 5.8 Power Flow Modeling Discussion

Power flow on transmission projects is subject to limits based on the accumulated capacity. In addition power flow follows physical laws that are commonly modeled by static AC power flow or DC power flow formulations. DC power flow is less accurate than the AC model, but in the context of long term planning, it is typically considered accurate enough. The AC power flow equations are nonlinear with respect to power variables, while the DC power flow equations are linear, so the DC model is typically preferred for long term planning.

Investment in transmission capacity on an arc from one load zone to another changes the total impedance and admittance in that arc and therefore

introduces quadratic expressions in the DC power flow equation determining the flow on that arc, making the overall optimization model a problem of mixed integer quadratically constrained optimization (MIQCP). MIQCP solvers do exist but they cannot handle models of the size resulting from grid capacity expansion in practical applications. Therefore, we consider several relaxations and reformulations of the DC power flow model.

First, if the DC power flow constraints are removed entirely and no further constraints are placed on transmission flow other than bounds, then the resulting model is the *transportation model*.

Second, if the DC power flow constraints are applied only to flows on existing capacity, and retirements of existing capacity are not allowed, then the relevant impedance and admittance parameters remain constant, so no nonlinear expressions are introduced. The resulting model is called the *hybrid model* as it is the DC model on existing capacity and the transportation model on new capacity.

Third, if capacity investments are permitted only in whole numbers of units, then the quadratic DC power flow equation can be reformulated as linear constraints using integer variables and the disjunctive constraint (Big-M) technique. The integrality constraints can be relaxed for computational performance with the loss of a guarantee of accuracy, but it is no less accurate than the hybrid model. The resulting model is the *disjunctive model*.

## 5.9 Transmission Power Bounds

All transmission projects are subject to bounds limiting the flows to the accumulated capacity:

$$|p_{it}| \leq c_{iy}^{acc} \quad \forall i \in I^{tr}, y \in Y, t \in T_y \quad (45)$$

## 5.10 Transportation Model of Power Flow

The transportation model of power flow consists of the power flow bounds (45) and no further constraints.

## 5.11 Quadratic DC Power Flow

The DC power flow model includes a product term with the number  $z_{iy}^{acc}$  of operating units times voltage angles  $\theta_{nt}$ :

$$p_{it} = -A_i^{bdc} z_{iy}^{acc} (\theta_{nt} - \theta_{n't}) \quad \forall i \in I^{tr}, y \in Y, t \in T_y, n = n_i^{fr}, n' = n_i^{to} \quad (46)$$

## 5.12 Linear DC Power Flow on Existing Transmission Capacity with no Retirement

If retirement of existing transmission projects is not allowed, then the quadratic constraints (46) can be replaced by linear constraints for those projects, since the number of units is fixed to the scheduled value:

$$p_{it} = -A_i^{bdc} Z_{iy}^{sch} (\theta_{nt} - \theta_{n't})$$

$$\forall i \in I^{tr} \cap I^{ext}, y \in Y, t \in T_y, n = n_i^{fr}, n' = n_i^{to} \quad (47)$$

$$z_{iy}^{ret} \leq 0 \quad \forall i \in I^{tr} \cap I^{ext}, y \in Y \quad (48)$$

### 5.13 Disjunctive DC Power Flow on New Transmission Capacity

For new transmission projects, or if retirements are allowed, the quadratic DC power flow constraints (46) can be modeled with a disjunctive, or big-M, formulation. If  $z_{iy}^{acc}$  is required to be an integer, then this big-M formulation is equivalent to (46). If  $z_{iy}^{acc}$  is bounded by 1, then this formulation is straightforward:

$$|p_{it} + A^{bdc} (\theta_n - \theta_{n'})| \leq M (1 - z_{iy}^{acc})$$

$$\forall i \in I^{tr} \cap I^{new}, \forall y \in Y, t \in T_y, n = n_i^{fr}, n' = n_i^{to} \quad (49)$$

$$z_{iy}^{acc} \leq 1 \quad \forall i \in I^{tr} \cap I^{new}, y \in Y \quad (50)$$

The big-M formulation can be generalized to  $z_{iy}^{acc} > 1$ , but we have not implemented this in the current version of GridEO.

### 5.14 Hybrid Power Flow Formulation

The hybrid formulation uses the transportation model for new transmission projects and the linear DC power flow model (47,48) for existing transmission projects.

### 5.15 Power Balance

System-wide power balance is formulated with slack variables for over- and under-generation:

$$\sum_{i \in I^{gn}} p_{it} + p_t^- = P_t^{ld} + p_t^+ \quad \forall n \in N, \forall t \in T \quad (51)$$

If a transmission network is considered, then zonal power balance is used, with zonal slack variables:

$$\sum_{i \in I^{gn} \cap I_n} p_{it} + \sum_{i \in I^{tr} \cap I_n^{to}} p_{it} + p_{nt}^- = \sum_{i \in I^{tr} \cap I_n^{fr}} p_{it} + p_{nt}^+ + P_{nt}^{ld}$$

$$\forall n \in N^{ld}, t \in T \quad (52)$$

The slack variables are nonnegative:

$$p_t^+ \geq 0 \quad \forall t \in T \quad (53)$$

$$p_t^- \geq 0 \quad \forall t \in T \quad (54)$$

$$p_{nt}^+ \geq 0 \quad \forall n \in N^{ld}, t \in T \quad (55)$$

$$p_{nt}^- \geq 0 \quad \forall n \in N^{ld}, t \in T \quad (56)$$

Practically, the slack variables are only needed to cover the load:

$$p_t^+ \leq 0 \quad \forall t \in T \quad (57)$$

$$p_t^- \leq P_t^{ld} \quad \forall t \in T \quad (58)$$

$$p_{nt}^+ \leq 0 \quad \forall n \in N^{ld}, t \in T \quad (59)$$

$$p_{nt}^- \leq P_{nt}^{ld} \quad \forall n \in N^{ld}, t \in T \quad (60)$$

If system-wide power balance is used, then the zonal slacks are fixed to 0, and similarly, if zonal power balance is used, then the system-wide slacks can be fixed to 0.

## 5.16 Investment Policy - Planning reserve

Initially we are implementing a linear planning reserve constraint. In future work we will implement a piecewise linear effective load carrying capacity (ELCC) approach. The planning reserve constraint requires that the total weighted capacity in each planning reserve zone exceeds a given requirement:

$$\sum_{i \in I_n} A_i^{nqc} c_{iy}^{acc} \geq C_{ny}^{pr} \quad \forall n \in N^{pr}, y \in Y \quad (61)$$

## 5.17 Operational Policy - Carbon Cap

The carbon cap is enforced as a soft constraint:

$$\sum_{i \in I_n, t \in T_y} D_t A_i^{gci} A_i^{hr} p_{it} \leq D_y (G_{ny}^{cc} + g_{ny}^{cc+}) \quad \forall n \in N^{cc}, y \in Y \quad (62)$$

Violation of the carbon cap is modeled as a nonnegative variable, which appears in the objective with a penalty coefficient:

$$g_{ny}^{cc+} \geq 0 \quad \forall n \in N^{cc}, y \in Y \quad (63)$$

The total carbon cap penalty incurred is

$$w^{cc} = \sum_{y \in Y, n \in N^{cc}} A_y^{df} F_{ny}^{ccp} D_y g_{ny}^{cc+} \quad (64)$$

## 5.18 Operational Policy - Carbon Tax

The carbon tax is modeled as a penalty coefficient on the carbon emission, which is determined by the carbon intensity of the generation, resulting ultimately in a cost coefficient on power generation. The total carbon tax incurred is

$$w^{ct} = \sum_{y \in Y, t \in T_y, n \in N^{ct}, i \in I_n} F_{ny}^{ctr} D_t A_i^{gci} p_{it} \quad (65)$$

## 5.19 Operational Policy - Energy Target

Energy target constraints require that the total energy generation in a given energy target zone over a given horizon exceed a prescribed fraction of the total load in certain associated load zones. The energy target is enforced as a soft constraint:

$$\sum_{i \in I_n, t \in T_h} D_t p_{it} + \sum_{t \in T_h} D_t p_{nh}^{et+} \geq A_{nh}^{et} \sum_{n' \in N_n^{etld}, t \in T_h} D_t P_{n't}^{ld} \quad \forall n \in N^{et}, h \in H_n^{et} \quad (66)$$

Constraint violations are nonnegative if allowed and otherwise equal to 0:

$$p_{nh}^{et+} \geq 0 \quad \forall n \in N^{et} \cap N^+, h \in H_n^{et} \quad (67)$$

$$p_{nh}^{et+} = 0 \quad \forall n \in N^{et} \setminus N^+, h \in H_n^{et} \quad (68)$$

Constraint violations appear in the objective with a penalty coefficient:

$$w^{et} = \sum_{n \in N^{et}, h \in H_n^{et}} F_{nh}^{et} p_{nh}^{et+} \quad (69)$$

If we use the investment periods instead of energy target horizons, we have:

$$\sum_{i \in I_n, t \in T_y} D_t p_{it} + \sum_{t \in T_y} D_t p_{ny}^{et+} \geq A_{ny}^{et} \sum_{n' \in N_n^{etld}, t \in T_y} D_t P_{n't}^{ld} \quad \forall n \in N^{et}, y \in Y \quad (70)$$

$$p_{ny}^{et+} \geq 0 \quad \forall n \in N^{et} \cap N^+, y \in Y \quad (71)$$

$$p_{ny}^{et+} = 0 \quad \forall n \in N^{et} \setminus N^+, y \in Y \quad (72)$$

$$w^{et} = \sum_{n \in N^{et}, y \in Y} A_y^{df} F_{ny}^{et} D_y p_{ny}^{et+} \quad (73)$$

For simplicity, we can opt for a hard constraint version:

$$\sum_{i \in I_n, t \in T_y} D_t p_{it} \geq A_{ny}^{et} \sum_{n' \in N_n^{etld}, t \in T_y} D_t P_{n't}^{ld} \quad \forall n \in N^{et}, y \in Y \quad (74)$$

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