A Figure-of-Merit for Beta Cell Detector Characterization

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1.0 Introduction

In order to decrease the minimum detectable activities (MDAs) of beta-gamma radioxenon detectors, it is important to increase the ability to resolve the individual isotopes. One proposed method for doing this is to increase the energy resolution of the beta cell through the use of silicon detectors. While silicon detectors can improve the energy resolution, it is accompanied with a decrease in detection efficiency compared to plastic scintillator beta cells. Due to the uncertainty on the impact of the competing variables, we have developed a figure-of-merit (FOM) capable of determining the impact of detector parameters on the MDAs. By utilizing the FOM to analyze different detectors, we are able to directly compare current and future detectors and estimate their impact on the radioxenon MDAs.

2.0 Simulation Techniques

The beta-gamma simulation tool (BGSim) developed by PNNL was utilized throughout this work in order to study the effect of varying parameters of interest. For these simulations, an 11-hour count time was utilized, with an assumed xenon volume of 1.3 cm$^3$. The background count rate was 0.24 events per minute, and the associated live time was taken to be 100% of the count time. With the simulated spectra, the regions-of-interest (ROI) were determined to be the full width at $1/15$th of the maximum. This allowed for consistent ROI criteria regardless of the changing ROI size due to the variation in detector energy resolutions. After the new ROIs were determined, the calibration data sets were processed through the PNNL developed beta-gamma Calibration (BGCal) software to perform the efficiency calibrations and obtain the interference ratios. Throughout this report, we focus on the relative behavior of the MDA as impacted by detector parameters, so the units are not of importance (scale is given in mBq as a point of reference).

3.0 Parameters of Interest and their Impact on the MDC

3.1 Detection Efficiency

The primary contributor to the variation in detection efficiency is the geometric efficiency of the beta cell. For current generation beta cells, the plastic scintillator covers nearly a $4\pi$ solid angle. For silicon detectors, the detector covers an area closer to $2\pi$. Since geometric efficiency is based on the interaction probability, it is independent of the emission of a beta continuum and a conversion electron peak. For this reason, we focus on the impact to the $^{133}$Xe MDA with respect to the geometric efficiency. The impact of the geometric efficiency is seen in the measured efficiency during the calibration process, and can be varied within the .PHD file. As the detection efficiency is decreased from 100 percent, the MDA is inversely proportional to the detection efficiency percent. This is to be expected since the equation for calculating the MDA is inversely proportional to both the branching ratio and the beta-gamma efficiency.
3.2 Energy Resolution

Energy resolution has the largest impact on the MDA of the xenon isomers ($^{133m}$Xe and $^{131m}$Xe). As the energy resolution improves, ROI becomes smaller, thus reducing the backgrounds both from the detector and from $^{133}$Xe. In this section, we focus on the impact of the decreased background counts due to the variation in energy resolution. Since the beta continuum is a broad energy spectrum, the improved energy resolution has little impact on the MDA for $^{133}$Xe and $^{135}$Xe, Figure 2. Figure 2 appears to show a large amount of scatter due to the scaling and the inherent decoupling of the energy resolution and the impact on the MDA. For the isomers, we see a direct linear relationship between the decrease in energy resolution and a decrease in MDA due to the decrease in background counts, Figure 3.
Figure 2. $^{135}$Xe MDA as a function of energy resolution, the variation is within the statistical variance of the analysis.

Figure 3. $^{133m}$Xe MDA as a function of energy resolution.
3.3 **Interference Ratios**

As the extent of the ROI windows for $^{131m}$Xe and $^{133m}$Xe decreases in size with improved energy resolution, the localization of the metastable signals increases along with reduced background interferences from the beta continuum. The impact of the variation in energy resolution was studied for the case of only $^{133}$Xe present. An activity concentration of 1.0 mBq/m$^3$ of $^{133}$Xe was simulated, and the subsequent data was analyzed for the respective MDA of $^{131m}$Xe and $^{133m}$Xe. Figure 4 shows the respective MDAs as a function of beta cell energy resolution. While this figure also includes the MDA component due to the detector background, we feel it important to account for the impact separately due to the benefits gained from the discriminating power of smaller interference ratios.

![Figure 4](image)

Figure 4. $^{131m}$Xe and $^{133m}$Xe MDA as a function of energy resolution with a 1 mBq activity of $^{133}$Xe present

Another important result of the decreased interference ratio variation is the MDA of the isomers as a function of $^{133}$Xe activity. Figure 5 shows the MDA for both $^{131m}$Xe and $^{133m}$Xe as a function of $^{133}$Xe activity for an energy resolution of 10 keV. Both curves are fit with a power law, with the MDA being approximately related to the $^{133}$Xe activity through the relationship of activity to the power of 0.64. The coefficient varies with energy resolution, but the trend is consistent throughout. Since the trend is consistent for varying energy resolutions, we focus on the variation of the coefficient term, which is encompassed in Figure 5.
3.4 Memory Effect

Memory effect can affect the MDA in two manners: the percent memory effect impacts the likelihood that the detectors will be blinded for a subsequent sample if a hot measurement is made (such as Fukushima). For the purpose of the FOM, we will focus on the impact of the memory effect on the measurement duration. If the memory effect is not present, then there is no gas background needed, and the sample count can be counted for an extended time, e.g., 24 hours. In this case, the MDA should decrease by a factor of $\sqrt{2}$ to account for the increased number of counts within the detector. One practical case where this comparison is important is when coating plastic scintillator cells to prevent memory effect results in a decrease in energy resolution. If the MDA for the 12-hour run at the initial energy resolution is better than that of the 24-hour run at the final energy resolution, then that process is not adequate for performing the memory-effect coatings\(^7\). Figure 6 shows the MDA for $^{131m}$Xe and $^{133m}$Xe as a function of energy resolution in the presence of 1 mBq/m\(^3\) of $^{133}$Xe for 11 and 22 hour count times. The impact of memory effect on the FOM is analyzed in the following section.

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Figure 5. MDA of $^{131m}$Xe and $^{133m}$Xe as a function of $^{133}$Xe activity for an energy resolution of 10 keV
4.0 Generalized Figure-of-Merit

With the parameters studied in the previous section, we have developed a figure-of-merit (FOM) for the purpose of directly comparing the effectiveness of detectors with different operating conditions. In general, the FOM can be written as:

$$FOM = \frac{(\text{Detection Efficiency})(\text{Res. Impact on Backgrounds})(\text{Res. Impact on Interferences})}{\text{Memory Effect Count Time}}$$

where Res. stands for resolution. In contrast to some more conventional FOM’s, in which larger numbers indicate higher performance, we have selected to go with the opposite indicator. For the FOM that we have developed, the lower the number, the better the performance. We chose this format in order to more closely align with the minimum detectable activity (MDA), which is the primary parameter of interest when determining a detector’s capability. While the FOM, will not be an exact indicator of the MDA for a simulated detector, it will give the user an idea of what results can be expected. In order to weight each of the parameters correctly, we set 0.1 mBq as the MDA limit for a system with 100% detection efficiency, 30 keV FWHM energy resolution, and a 12 hour count.

Since the interference ratio impact depends on both the energy resolution and the $^{133}$Xe activity, we standardize to 1 mBq/m$^3$ of $^{133}$Xe activity for the FOM study. Taking the fits from the previous sections, and the normalized values, we obtain the following dependences:

$$\text{Detection Efficiency} = \text{Efficiency}^{-1},$$

\[
\begin{align*}
(11\text{hr Xe-133}) \text{ Xe-133m} & : f(x) = 0.0035667x + 0.063056, \quad \text{RMSE} = 0.0062552, \quad R^2 = 0.98585 \\
(22\text{hr Xe-133}) \text{ Xe-133m} & : f(x) = 0.0027333x + 0.059444, \quad \text{RMSE} = 0.0070598, \quad R^2 = 0.96981
\end{align*}
\]
where a detection efficiency of 100% gives a value of 1. As the detection efficiency decreases, the FOM and MDA increase proportionally.

\[
\text{Resolution Impact on Backgrounds} = (\text{Res. [keV]} * 0.0025 + 0.025),
\]

where the MDA is proportional to the energy resolution multiplied by 0.0025 with an offset of 0.025 needed to obtain an MDA of 0.1 mBq for an energy resolution of 30 keV, see Figure 3.

\[
\text{Resolution Impact on Interference Ratios} = (\text{Res. [keV]} * 0.003 + 0.01),
\]

where the MDA is proportional to the energy resolution multiplied by 0.003, with an offset of 0.01 required to obtain an MDA of 0.1 mBq for an energy resolution of 30 keV and a $^{133}\text{Xe}$ activity concentration of 1 mBq/m$^3$ in 1.2 cc of xenon, see Figure 4.

\[
\text{Memory Effect Count Time} = \sqrt\frac{\text{Count Time}}{12 \text{ [hrs]}},
\]

where the effect of an increased or decreased count time is compared to the baseline of a 12 hour count for SAUNA systems.

Combining each of the factors, we obtain a FOM as given by:

\[
FOM = \frac{100 (\text{Efficiency [\%]} )^1 (\text{Res. [keV]} * 0.0025 + 0.025)(\text{Res. [keV]} * 0.003 + 0.01)}{\sqrt{\frac{\text{Count Time}}{12 \text{ [hrs]}}}}.
\]

Using this FOM, we compare 6 different detector architectures: a plastic beta cell, coated beta cell to remove memory effect (24 hour count), a coated beta cell with reduced memory effect (12 hour count), the PIPSBox (a French designed pancake style silicon beta cell)$^3$, a silicon beta cell with higher efficiency but still containing plastic and memory effect, and an optimal silicon beta cell. The parameters used for comparison and the resulting FOM are given in Table 1.
Table 1. FOM calculated for detector setups and parameters of interest. A lower FOM indicates a better performing detector.

<table>
<thead>
<tr>
<th>Detector</th>
<th>Efficiency</th>
<th>Resolution</th>
<th>Memory Effect</th>
<th>Count Time</th>
<th>FOM</th>
</tr>
</thead>
<tbody>
<tr>
<td>Plastic Beta Cell</td>
<td>100%</td>
<td>30 keV</td>
<td>Yes</td>
<td>12 hours</td>
<td>0.2</td>
</tr>
<tr>
<td>Coated Plastic Beta Cell</td>
<td>100%</td>
<td>40 keV</td>
<td>No</td>
<td>24 hours</td>
<td>0.18</td>
</tr>
<tr>
<td>Coated Plastic Beta Cell</td>
<td>100%</td>
<td>40 keV</td>
<td>Minimal</td>
<td>12 hours</td>
<td>0.255</td>
</tr>
<tr>
<td>PIPSBox</td>
<td>50%</td>
<td>10 keV</td>
<td>No</td>
<td>24 hours</td>
<td>0.128</td>
</tr>
<tr>
<td>Si PIN Beta Cell</td>
<td>67%</td>
<td>10 keV</td>
<td>Yes</td>
<td>12 hours</td>
<td>0.134</td>
</tr>
<tr>
<td>Optimal Si Beta Cell</td>
<td>100%</td>
<td>10 keV</td>
<td>No</td>
<td>24 hours</td>
<td>0.064</td>
</tr>
</tbody>
</table>

As can be seen from the above table, if a memory effect coating can remove the memory effect completely and allow for a removal of the gas background count, the FOM will improve, even with a 10 keV loss in resolution. It is clear, however, that the FOM will improve dramatically from the improved energy resolution of a silicon detector, but keeping the geometric efficiency close to 100% is still a high priority. As a point of reference, we have included the case of an optimal cell where we obtain silicon detector energy resolution while maintaining the geometric efficiency of the current plastic scintillator beta cells. In this case, the FOM improves by approximately a factor of three over the current generation of cells, suggesting that there would be approximately a three-fold reduction in the MDA.

5.0 Conclusions and Future Work

This work has led to the development of a FOM that enables rapid comparison of beta-cell detectors of various materials and form factors to estimate and compare the performance of current and future beta cells. This FOM, however, does not include other potential detection techniques including triple coincidence detection. To determine the added benefit of other detection features will require additional simulation and experimental testing and validation of the reduction in MDA. If a $^{133}$Xe activity-specific FOM is desired, then the FOM should be updated to account for the results shown in Figure 5. While this is possible, we do not feel it is needed, since the FOM is designed to provide an overall characterization of a detector, and not be dependent on the activity of the sample measured.
6.0 References


