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DP Chassin

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Prepared for the U.S. Department of Energy
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Summary

Recent work applying statistical mechanics to economic modeling has demonstrated the effectiveness of using thermodynamic theory to address the complexities of large-scale economic systems. Transaction-based control systems depend on the conjecture that when control of thermodynamic systems is based on price-mediated strategies (e.g., auctions, markets), the optimal allocation of resources in a market-based control system results in an emergent optimal control of the thermodynamic system. This report proposes an abstract machine model as the necessary precursor for demonstrating this conjecture and establishes the dynamic laws as the basis for a special theory of emergence applied to the global behavior and control of complex adaptive systems. The abstract machine in a large system amounts to the analog of a particle in thermodynamic theory. This permits the establishment of a theory dynamic control of complex system behavior based on a strict analog to statistical mechanics. Thus, we may be better able to engineer systems having few simple control laws for a very small number of devices types, which when deployed in very large numbers and operated as a system of many interacting markets, yield the stable and optimal control of the thermodynamic system.

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1.0 Introduction

The idea of controlling a complex engineered system using one or more market-like processes is not new, nor has it always been done with a literal market in the sense that price and quantity are the joint means of determining the allocation of a scarce resource. Indeed, in the early 1980s, so-called contract networks were demonstrated to allocate scarce central processing unit (CPU) time to competing tasks in computers (Smith 1980). Nevertheless, the challenges faced by system designers who wish to use market-based control strategies are daunting. Such systems are generally called complex adaptive systems because they exhibit a property that is regarded as a profound obstacle to crafting stable and robust classical control strategies: emergent behavior (Kaufman 2000). This is the property of systems that causes them to exhibit global behaviors not allowed for in the control strategy. For example, building heating, ventilation, and air-conditioning (HVAC) systems exhibit a phenomenon called global hunting, an common artifact of the control strategies employed to govern the thermodynamic process (Karahara et al. 2001, Matsuba et al. 1998, Krakow 1998). Many other engineered systems (e.g., power grids, military command / control / communications / intelligence systems, air traffic control systems, biological systems) exhibit this property and are thus candidates for membership in the class of systems characterized by complex adaptive behavior (Loyd 1995).

2.0 The Abstract Transactive Machine Model

It is with the object of understanding the relationship between the rules governing the behavior of devices and the emergent behaviors of the systems that we set forth to establish a theory of control that applies to complex adaptive systems. While the full development of such a theory is beyond the scope of this report, it is necessary to devise a rigorous model for the constituent devices in such systems. We expect that having devised such a rigorous model, it will be possible to postulate laws, and ultimately develop definitions of average properties of complex systems that will be experimentally verifiable.

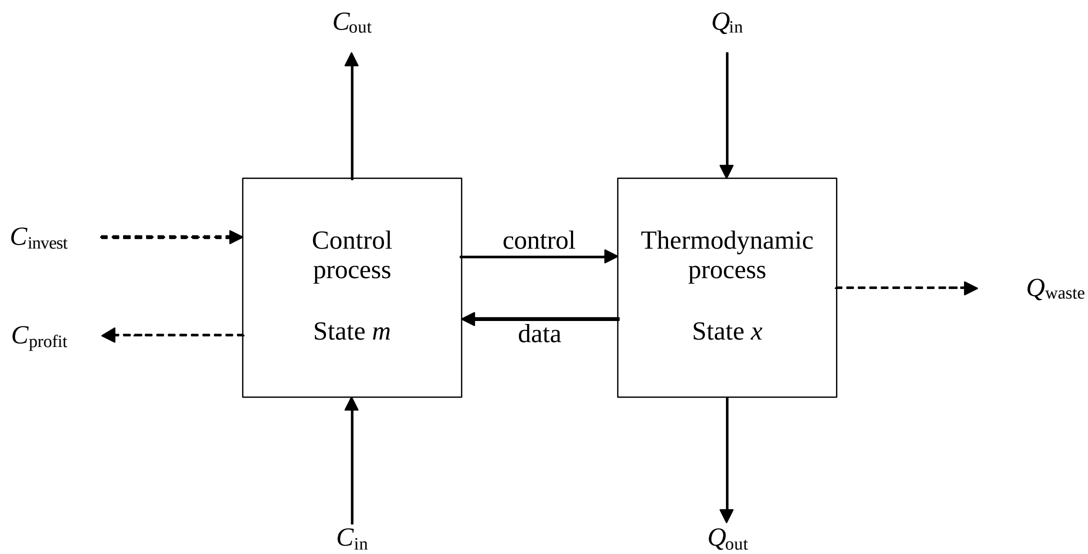

Figure 1: Abstract transactive machine

To better understand the discussion that follows we must first establish basic concepts regarding the complex adaptive systems we shall consider. Thus, we begin by defining an abstract machine[a], the basis for all devices we can conceive in real systems. Figure 1 illustrates the abstract machine we shall use henceforth in all considerations. The abstract transactive machine is intended to provide the basis for coupling a thermodynamic work process to an economic control process. The specifics of both these processes are not important to the discussion and will

[a] In the context of this theory the device is meant to include both the machinery that performs the thermodynamic process and the control logic that in some fields is called the controller and in others is referred to as the agent.

be left to future discussions on the development of rules and the resulting system behavior. The abstract transactive machine is defined thus:

> «A transactive machine is a device that converts in a time t a quantity $Q_{in}t$ of resources obtained from another transactive machine at a cost $C_{out}t$ and produces a quantity $Q_{out}t$ of resource sent to another transactive machine for which it receives compensation $C_{in}t$.»

The device may also involve the production of a waste at a rate Q_{waste}, for which no consideration can be derived and the device may require the investment of capital at a cost $C_{\text{invest}}t$ or produce $C_{\text{profit}}t$, all of which play a role in the basis for the device's value. A number of variations on this model are possible, but fundamentally every conceivable engineered device operating in an economic system should be based on this abstract design.

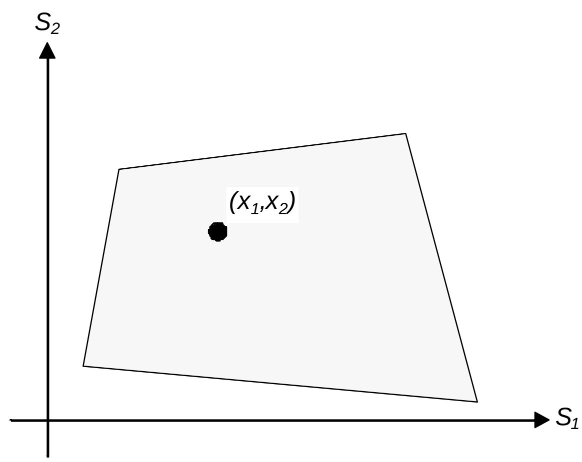

Figure 2: Device state space and operational envelope

A device state may be represented in a N-dimensional space, where the states x_1 through x_N of the device's resources are measured on the axes. The device's inherent properties restrict it to a region of this space, its operating envelope. This envelope may be one dimensional, i.e., the device may alter the state of only one particular resource. The device's control logic must not negotiate contracts that will require the device to be operated outside that envelope. In addition, the control logic may choose to implement strategies that further constrain the space for a variety of reasons, which are not important to the purposes of this discussion.

The device must be a deterministic machine in the sense that at any given instant, if a state change in the thermodynamic process occurs that is communicated to the control logic, the control logic acting upon the thermodynamic process will select any new state for the machine in no greater than a polynomial time with respect to the number of possible state sensed[a]. Therefore, given a sufficient settling time, the device will always be in a single known state chosen from among a finite and unchanging set of states. The term unchanging is used only in the sense that the number of accessible states does not change perceptibly with respect to the time scale in which a device's state may be observed. There is no requirement that the set of possible states remain unchanging for all time, only for the duration of observations and actions of the control logic.

We should also note an important consideration regarding the state of a device. The control logic of the device may in fact decide to completely disconnect the device from a system, i.e., reduce all rates to zero. In another case, devices may be created and destroyed according to rules regarding their currency balance. This has the effect of removing devices from existence in the system. In this sense, we must be use caution when considering the number of devices in a system. The number of devices may not be constant depending on the characteristics of the devices involved. To use an analogy to thermodynamics, an ideal gas has a constant number of particles and thus equations of state where energy is a linear function of temperature. In contrast, a photon gas has a variable number of particles and the internal energy function is volume dependent. Thus we must consider the possibility that systems in which devices may turn off are perhaps phenomenological distinct from systems in which all devices must always have at least one non-zero rate and a non-zero currency balance. The only case in which such a distinction would be unnecessary would be if "off" state of a device has a non-zero potential. This would indeed be true for example, if turning a device off had value in a system, such as when underfrequency load shedding is used in a remedial action scheme to maintain frequency stability on power grids (Grigsby 1999).

[a] This is the most important criterion for Turing determinism as it applies in the context of this discussion. If the sensor data from the thermodynamic process has a finite number of states (which in digital control systems it necessarily does), then the control state sent to the process must be determined by the controller in at most a time that is a polynomial function of the number of sensor states. This assures us that the device will always be a correct state, or in a transient state en route to a correct state that is reached in a finite time.

3.0 Interactions Between Machines

Having defined a single device, we must now define a transaction, the only known interaction between two devices. During a transaction, two devices determine the price and quantity for the trade of a resource for currency (*negotiating* the contract), and then perform the exchange (*executing* the contract). There are certain extended types of transactions that permit delaying the exchange, or making the exchange conditional, *future* and *contingent* contracts, respectively. However, for the purpose of this discussion, we will not address these and focus exclusively on the dynamics of *immediate* contracts. Figure 3 illustrates the two steps in which a contract is negotiated and executed.

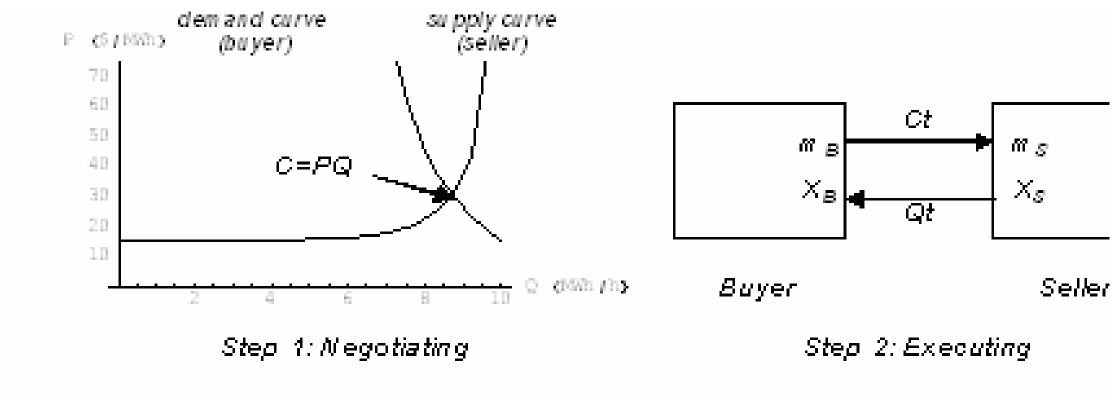

Figure 3: Transaction resolution by negotiation and execution

During the first step, an exchange of information takes place during which two essential values are determined: the price *P* at which the trade will occur and the quantity *Qt* of the resource to be traded. The method of determining these values is not predetermined, although one has been shown here to illustrate how it might be done. An initial price and quantity (*P*, *Q*) is offered or requested, for which a request or an offer is made, and continuing until a final, or closing price and quantity (*P*, *Q*) is determined and the first phase is concluded. During the second phase, the agreed upon trade is actually performed and the closing values of (*P*, *Q*) are in fact exchanged. This type of trade is called an immediate contract. There are two other types of contracts that may also be negotiate: future and contingent. In a future contract, the time at which the trade will be executed is negotiated. In contingent contracts, the conditions under which the trade will be executed are also negotiated.

4.0 Properties of Abstract Machines

Having defined devices, state spaces, and transaction spaces, we must identify the properties of devices and the fundamental relationships between them. The purpose of these definitions is not to establish a robust analogy to any known laws of economics or physics. Rather they create the special conditions needed to easily calculate the average properties of very large systems of devices interacting in the manner described above. For this reason, many of the analogies may seem over-constrained or even strained, but they are needed to ensure the completeness of later considerations and to allow a strict analogy to the conditions extant in thermodynamic theory. With this special circumstance in mind, we first define *transaction rate* with respect to a state[a] x of resource S, which is equivalent to consumption or production, depending on the sign. Thus:

$$dx = Q\, dt$$

in which dx is the *unit rate* of inflow or outflow of a resource, dt is *unit transaction duration* over which the resource was exchanged, and Q is the *resource rate* (which is analogous to velocity in mechanical systems) or quantity of resource involved in the transaction with respect to the resource S, the sign of which will depend on the direction of the transaction (negative for outflows and positive for inflows).

Next we define the concept of *device reserve*, which is the amount of money available to purchase resources, represented as m,. (This can be considered analogous to the mass of an object in mechanics. We shall see that as devices interact with other devices or in the market, those devices with large reserves behave as though they had high mass, and those with small reserves low mass. Naturally, devices that cannot maintain reserves will be observed to act as massless devices.) It should also be noted that devices may store a quantity of resources for a time. If a device maintains a strict balance of reserves and resources at the beginning and end of every transaction, the device is said to be an *elastic device*. Devices that can carry an imbalance of any resource or reserve for more than the duration of a single transactions is said to be an *inelastic device*. In transactive systems, the great majority of devices will be found to have elastic behavior. For the scope of this discussion, we will consider only elastic devices[b].
We observe that the change in cash reserves is the sum of the prices of k resources acquired, multiplied by the quantity of resource traded, i.e.,

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[a] Some devices may not have a well-defined state (e.g., stored capacity of electricity in a battery) of some or all their resources, in which case the coupling between input and output resources is very strong. The greater the storage capacity of a resource, the weaker the coupling of the resources. This will be discussed in more detail when device inertia is introduced.

[b] Do not confuse transaction elasticity in this sense with demand and supply elasticity, which will be discussed later. Transaction elasticity is analogous to elasticity in collisions in physical systems.

$$dm/dt = C = -\sum_{s=1}^{k} P_s Q_s$$

where P_s is the price at which the transaction for the resource S was made. Thus, we define *unit value* in general as

$$dm = C\, dt = -P\, Q\, dt$$

We define *device inertia* as the resistance offered by a device to changes in all its rates Q. Most devices' designs will produce natural couplings of resistances across resources. Often a change in the rate of one resource will result in changes in the rates of the other resources[a].
Having defined these transactions properties and the only intrinsic behavior of devices, we therefore postulate a definition of the only intrinsic property of a device that is conserved quantity in the absence of interactions with other devices. This is the *device momentum* with respect to the resource S represented as p, which is the product

$$p = m\, Q$$

As a result, we conclude that the change in device momentum with respect to a transaction is

$$dp/dt = dm/dt\; Q + m\; dQ/dt = P\, Q^2 + m\; dQ/dt = \frac{mQ^2}{r} + m\, dQ/dt$$

where $r = m / P$, i.e., the value of the device reserves relative to the price of the resource S.
We will therefore define that change in momentum as the *transactive force* of the device, F, as in

$$F \equiv dp/dt.$$

We should note that when two devises interact in a bilateral transaction, one becomes a buyer B and one is a seller S. Below, machine 1 is the buyer and machine 2 is the seller.

Through any arbitrary process the machines determine the competitive quantity and price of the transaction, which is the intersection the respective demand and supply curves of the machines.

[a] For example, a decrease in electric output from a generator results in a decreased fuel demand and an increased reserve capacity.

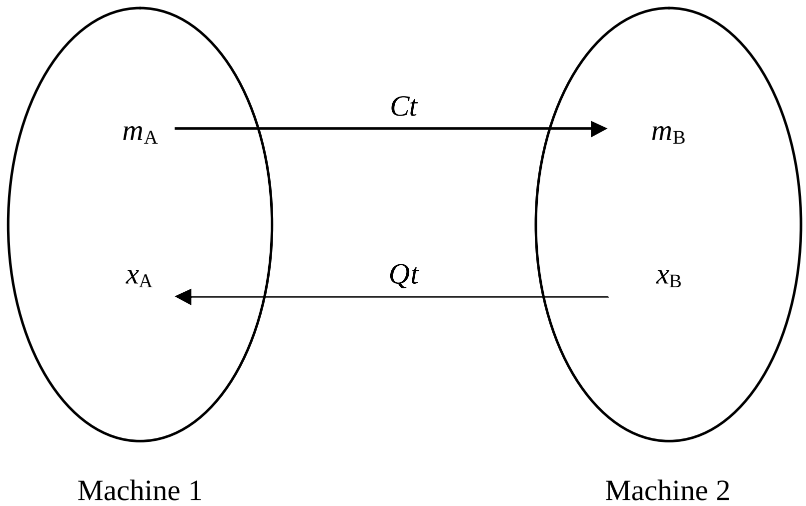

Figure 4: Abstract machines exchanging *Qt* and *Ct*

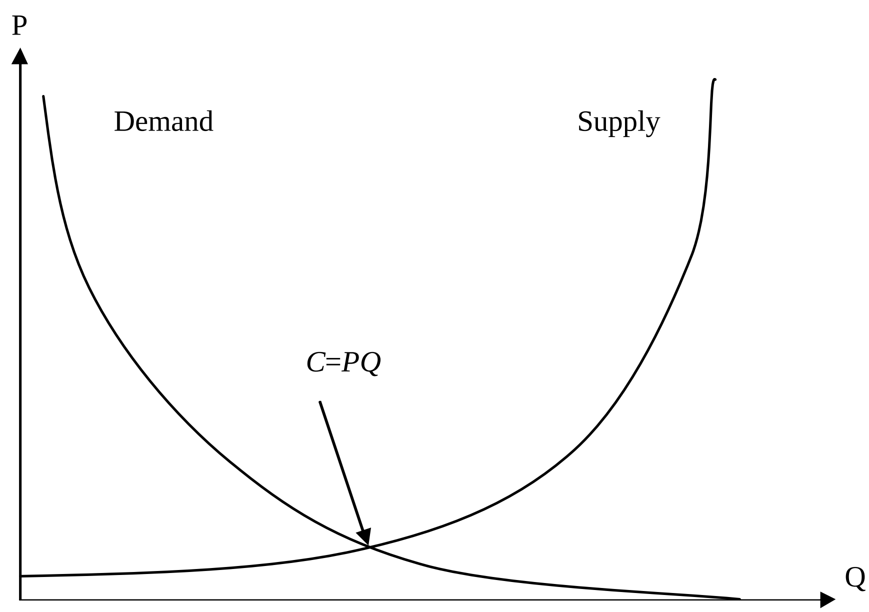

Figure 5: The competitive price and quantity

The momentum p is given by $p = mQ$, so we have

$$p_S = m_S Q$$
$$p_B = m_B Q$$

As they interact repeatedly, the change in momentum of each machine is opposite and equal. Thus

$$\frac{dp_S}{dt} = -\frac{dp_B}{dt}$$

or

$$\frac{dm_S}{dt}Q + \frac{dQ}{dt}m_S = -\frac{dm_B}{dt}Q - \frac{dQ}{dt}m_B \qquad (4.1)$$

because we know

$$C = \frac{dm_S}{dt} = -\frac{dm_B}{dt} \qquad (4.2)$$

we rearrange (4.1) and substitute from (4.2) to get

$$\left(m_B + m_S\right)\frac{dQ}{dt} = 0\,.$$

We conclude that when the total funds are non-zero, the system is at equilibrium and Q is unchanging over time. This is an important observation because it allows us to assert that the conservation of momentum and indeed the momentum postulate itself is consistent with a coherent interpretation of the equilibrium behavior of machines for any bilateral transaction. There is a special case of the market solution that can be obtained in closed form. If the supply curve for the seller is of the form

$$P_S = \frac{1}{\sqrt{1-\left(\frac{Q}{S}\right)^n}}$$

where S is the supply capacity of the system and the demand curve for the buyer is of the form

$$P_B = \left(\frac{D}{Q}\right)^{n/2}$$

where B is the demand of system, then we find that

$$Q = \left(D^{-n} + S^{-n}\right)^{-1/n}.$$

This approximation is useful for most transactive systems and will be used throughout this document.

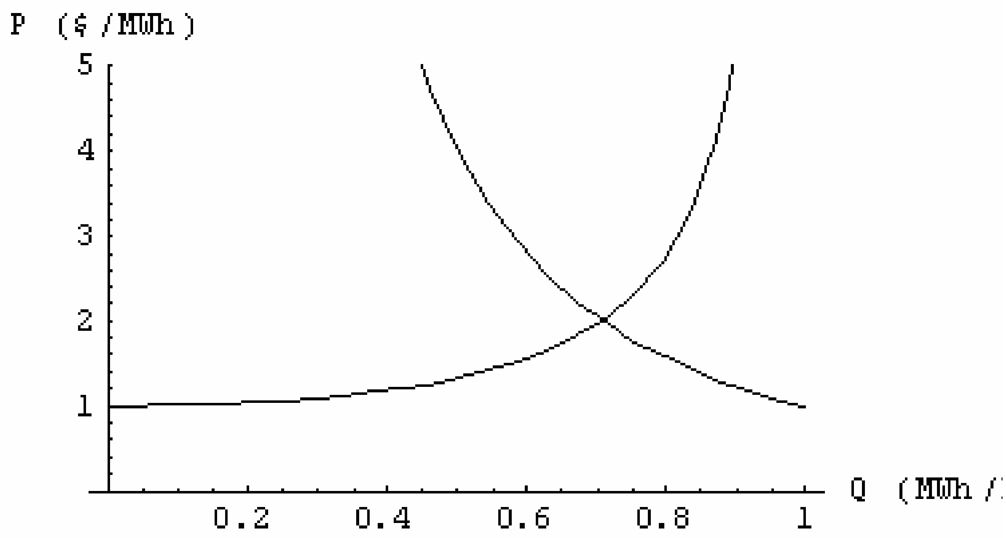

Figure 6: Approximate supply and demand curves

5.0 The Definition of Value

In the absence of interactions with other devices, a device has three distinct values that are collectively conserved. They are *potential value*, *transactive value*, and *intrinsic value.* The potential value is the sum of total cash reserves and trade value of the stockpiles of resources in the device. The transactive value represents the value realizable by a device through transactions with other devices. The intrinsic value is the value inherent in the internal structure of the device, such as the couplings of rates, rate limits, and other properties of the how the device may interact with other devices. Therefore, we have the *total value* or simply *value* of a device is:

$$U = U_{\mathrm{P}} + U_{\mathrm{T}} + U_{\mathrm{I}}$$

and the value U of a system of N devices is given by

$$U = \sum_{i=1}^{N} U_i$$

From these considerations we have defined a number of conservation laws. The first is that when devices conduct a transaction, they determine only the quantities P and Q, the actual price and quantity of a resource traded[a]. In addition, we expect all closed systems to conserve the total amount of resources available and the total cash available. Therefore, the total value of the system shall always remain constant, in spite of the fact that value may be partitioned in infinite ways between the potential and intrinsic values, as we shall see later. Finally, we assert that for any closed system of one or more devices not interacting with a market, the net change in momentum of the system is always zero.

[a] Therefore, any two unlike devices participating in a transaction can be expected to have different resulting values for their change in momentum because they may reasonably be expected to have different positions prior to engaging in the transaction.

6.0 Systems of Abstract Machines: Markets

Finally, we define a market. Markets represent the collective effect of devices that are participating in transactions for a certain resource. Thus, we have at least one market for each resource traded in a system of devices. If we refer back to Figure 3, we see that a buyer may start out trying to acquire as much quantity as possible while paying as little as possible. A seller then counters with a higher price. The buyer then rebuts with a different quantity and so on until they agree on both price and quantity. This process allows both the buyer and seller to come to some common understanding of what they collectively believe the price and quantity should be. This process is based on hidden information they each possess about themselves (e.g., how desirous of the resource the buyer is or how readily the seller can dispense it) and what their respective current positions and momenta are. This sense of mutual flexibility is the supply elasticity with respect to the seller and demand elasticity with respect to the buyer[a]. Markets serve to aggregate the collective elasticities of all the buyers and sellers for a given resource. In the context of real transactive control systems, we would wish to see that markets effectively determine the marginal price (i.e., the price of the next unit of resource) at a certain location in a system, called the *locational marginal price*. This price is usually based on the topological proximity of buyer to seller (which would include the effects of path congestion and/or losses), but this will not be easily considered until transport mechanism can be addressed in a more developed model. We shall call the effect of a market on transactions its *field*, the *lines* of which are based on the dimension of the resource, and the *strength*[b] of which is based on the collective transactive moments of those participating in interactions over that resource (see Figure 7).

We can see that devices and markets interact in various ways, depending on conditions. For example, a device selling a large quantity of resource entering the field of a highly supply-constrained market will be able to command a high price for its resource only so long as it does not significantly alter the supply elasticity of the market. By increasing supply to the market, the strength of the market's field is altered.

We will customarily work with two kinds of spaces, state space and transaction space. We will often consider the behavior of both devices and markets in both spaces, depending on circumstances. State space is used to describe the macroscopic properties (i.e., rates of inputs or outputs of resources) of a device and discern the effect of bilateral or market transactions on devices. Transaction space is used to describe the microscopic properties of transactions (i.e., the price of resources and the quantity exchanged in the transaction). Note that the macroscopic field

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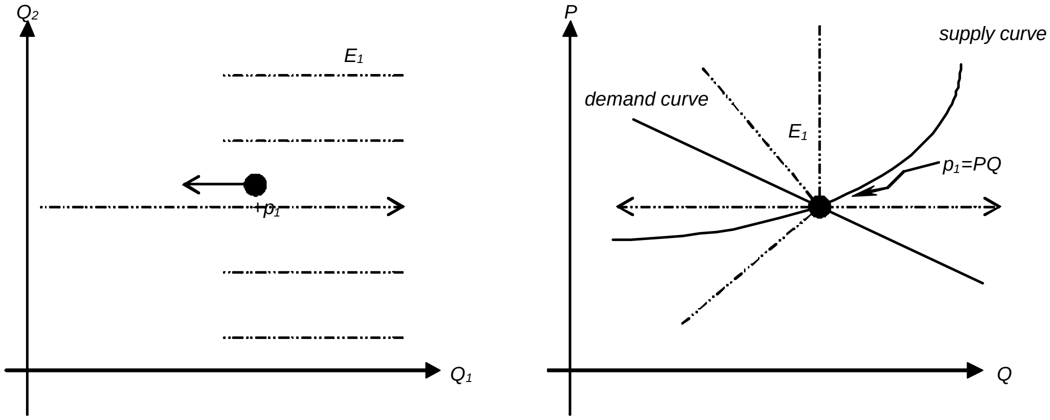
[a] The elasticities of the various resource rates of a device depend largely on the couplings between them, i.e., the device's inertia. However, the collective elasticity of a system depends on more than just the collective inertia because of the interplay between their inertia as they interact. Thus, the total elasticity of a system is also determined by the degree to which a device can interact to exchange elasticity of one resource for that of another.

[b] We would expect this quantity to be related to the inverse of the elasticity of the market with respect to the position of the device, buying or selling, on which it is acting. The question as to whether it can be related to the definition of elasticity in economics, i.e., $P/Q.dQ/dP$ remains unanswered at this time.



(a) State space *(b) Transaction space*

Figure 7: State space and transaction space.

of a market can be visualized as lines parallel to the axis along which the resource traded in that market is plotted, but as a point in the transaction space. Viewed in state space, markets cause devices to displace along the field lines and with respect to each other, while in transaction space they are attracted to the market's competitive point along lines radiating spherically. This visualization can be used to explain how system level behaviors are produced by what appears to be a hidden variable theory at the device level.

From the perspective of thermodynamic systems control, markets only make sense with respect to the topological locality of devices. However, locality for some types of systems can span large physical distances. For example, electric generation markets can span many thousands of kilometers. On the other hand, a market for lighting in a building may only span a few meters. We will assume that the systems under consideration are essentially non-local in nature, having hidden variable theories that govern the distant but correlated behaviors of devices (see Figure 8).

The effect of an executed transaction is to displace the two identical devices with respect to each other following symmetric vectors along the field of E. In the limit with no external consideration at play, if all available resources are utilized in the negotiation of a single trade, the seller's supply will drop to zero, and the buyer's cash will drop to zero leading to oscillatory behavior that is probably undesirable. However, if the trades are negotiated as a series of many small transactions in which only a small fraction of the total stock is traded, then the field of E will decrease slightly after each incremental transaction, causing the devices to approach each other until the field goes to zero. Therefore, for a correct solution, the transactions should be computed in small increments with respect to the stock of the devices until both devices have equal stock (given sufficient buyer cash or seller stock). At that point they will have exhausted their ability to trade any further and the potential value of any transaction will be zero.
It is interesting to note what occurs when a lone device has an excess or deficit of stock with respect to its own thermodynamic process. (Recall that the market field it creates is parallel to the axis of the resource.) When there is supply but no demand, the price is zero for any quantity.

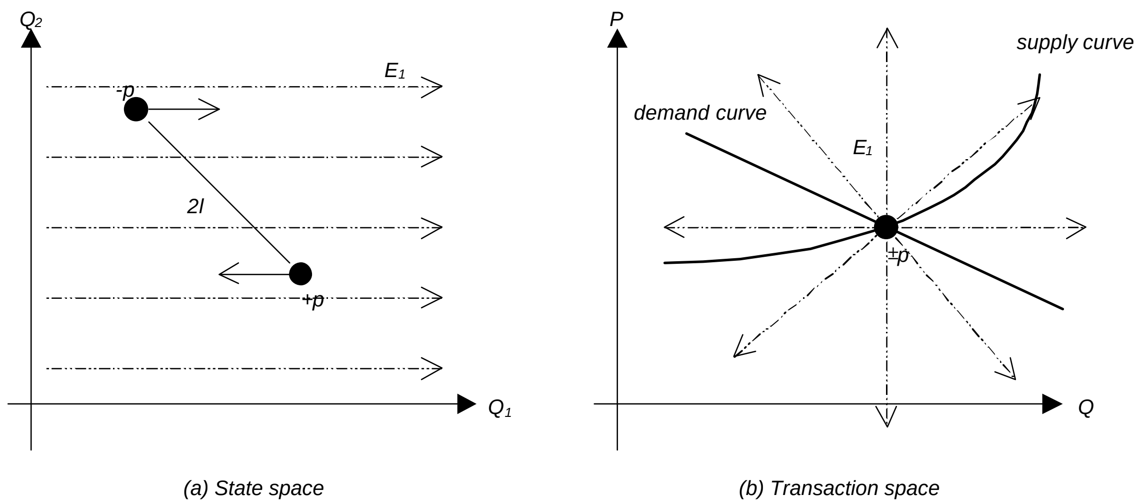


Figure 8: Effect of transaction on device

With a zero price, the transaction value is zero, regardless of quantity considered. Conversely, if there is a demand with no supply, then the price is infinite and the transaction value would be infinite were it not that the quantity is zero. Therefore, when a device is completely alone, it can only be in a single unchanging state, because *P* and *Q* must remain zero.

7.0 Conclusion

From these considerations we are able to deduce the existence of at least four laws that govern the behavior of abstract transactive machines and the systems in which they interact. They are:

Law 1

Every device has a state inertia, which is a joint function of the internal thermodynamic process and the control logic.

Law 2

The effect of a device on another device's inertia is a joint function of its inertia and the rates of its thermodynamic process.

Law 3

When two devices interact, they exchange equal and opposite values of resources and cash.

Law 4

The total value of a closed system remains constant.

Looking ahead, we expect other laws to be developed or postulated as we derive models for the average properties of systems of abstract machines. It is reasonable to expect that the analogy to statistical mechanics will be robust enough to enable a fairly straightforward derivation of properties analogous to temperature, chemical potential, kinetic potential, etc. It is in anticipation of the utility of these quantities that we assert the utility of the abstract transactive machine. Having the ability to describe average properties of complex adaptive systems that can be related to each other as properties of the system change is essential to devising control laws for devices that yield emergent and optimal control of the entire system.

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