West Valley Tank 8D-1 and 8D-2
Inventory Estimation Methodology

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Contents

1.0 Introduction ........................................................................ 1.1

2.0 Model of Radionuclide Inventory ............................................ 2.1

3.0 Tank 8D-1 and 8D-2 Mobile Inventory .......................................... 3.1
  3.1 Tank 8D-2 Mobile Waste Inventory Estimation ......................... 3.1
  3.1.1 Direct Tank 8D-2 Mobile Inventory .................................. 3.3
  3.1.2 Using Ratios to Complete the Tank 8D-2 Inventory ................ 3.5
  3.1.3 Propagation of Errors for Tank 8D-2 Inventory ...................... 3.5
  3.2 Tank 8D-1 Mobile Waste Estimation Methodology ................. 3.6
  3.2.1 Tank 8D-1 Inventory Estimation from CFMT Measurements and Zeolite Mass Estimates for Tank 8D-1 .................. 3.7
  3.2.2 Propagation of Errors for Tank 8D-1 Inventory ..................... 3.9

4.0 Tank 8D-1 and 8D-2 Fixed Waste Inventory .............................. 4.1
  4.1 Tank 8D-1 and 8D-2 Fixed Estimation Methodology ............... 4.1
  4.2 Propagation of Errors for Tank 8D-1 and Tank 8D-2 Fixed Inventory .............. 4.2

5.0 Summary ........................................................................... 5.1

Figures

2.1 Assumed Flow of Mobile Waste Component of Waste .................... 2.2
3.1 CFMT Sampling and Tank 8D-2 Transfer Plan for Tank 8D-2 Mobile Inventory Estimation ................................................................. 3.2
3.2 Schematic of Tank 8D-2 Mobile Waste Estimation ........................ 3.4
3.3 Schematic of Tank 8D-1 Mobile Inventory Estimation .................... 3.8
### Acronyms

<table>
<thead>
<tr>
<th>Acronym</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>CFMT</td>
<td>Concentrator Feed Make-Up Tank</td>
</tr>
<tr>
<td>Ci</td>
<td>Curies</td>
</tr>
<tr>
<td>LLW</td>
<td>Low-Level Waste</td>
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<td>HLW</td>
<td>High-Level Waste</td>
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<td>PNNL</td>
<td>Pacific Northwest National Laboratory</td>
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<td>PUREX</td>
<td>Plutonium/Uranium Extraction</td>
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<tr>
<td>THOREX</td>
<td>Thorium Extraction</td>
</tr>
<tr>
<td>WVDP</td>
<td>West Valley Demonstration Project</td>
</tr>
<tr>
<td>WVNS</td>
<td>West Valley Nuclear Services Company</td>
</tr>
<tr>
<td>WVSP</td>
<td>West Valley Support Project</td>
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</tbody>
</table>
1.0 Introduction

This report details work funded by the West Valley Support Project (WVSP) and the Tanks Focus Area Retrieval and Closure Program. The work was conducted by the Pacific Northwest National Laboratory (PNNL) and is in support of the West Valley Demonstration Project (WVDP). The WVDP site in New York was originally the site of a commercial nuclear fuel reprocessing plant. The high-level waste (HLW), approximately 2 million liters, produced during plutonium-uranium extraction (PUREX) and thorium extraction (THOREX) reprocessing campaigns at the plant and subsequent HLW preprocessing, was stored on site in three tanks identified as 8D-1, 8D-2, and 8D-4. Waste from the PUREX process was neutralized with NaOH for storage in a carbon steel tank designated as 8D-2. Neutralization resulted in a precipitated hydroxide sludge that settled to the bottom of the tank and was covered by a supernatant salt solution. The acidic THOREX waste, approximately 55,000 L, was first stored in a stainless steel tank (8D-4) and then added to the PUREX waste in Tank 8D-2. Supernatant decontamination, primarily cesium removal, was conducted by ion-exchange using in-tank columns suspended in Tank 8D-1. The cesium-loaded zeolite, resulting from the supernatant decontamination process, was dumped to the bottom of Tank 8D-1. Approximately 90% of the spent zeolite was transferred from Tank 8D-1 into Tank 8D-2 by the start of vitrification processing in June 1996. Periodically, the remaining spent zeolite contained in Tank 8D-1 is incrementally transferred to 8D-2. The combined waste in Tank 8D-2 continues to be processed through the Vitrification Facility into canisters for final repository disposal.

After the initial processing campaign of the HLW through the WVDP Vitrification Facility was completed, heels remained in the tanks. Additional waste retrieval efforts will continue to decrease the waste heels in the tanks. Final disposition of the residual waste and the waste tanks will require waste classification according to 10 CFR 61.55. Disposal options will depend on this classification. The WVSP is supporting waste characterization for final disposal by developing a methodology/model for interpreting current and future characterization data and how it relates to the tank inventory.

In this paper, a set of formulas is presented that estimates both the Tank 8D-1 and Tank 8D-2 radionuclide inventories, and their associated errors. The radionuclide inventory in each tank is assumed to be in two components: a mobile and a fixed waste component. Estimates for the mobile component of waste in each tank will use direct measurements from the Concentrator Feed Make-up Tank (CFMT). Estimates of the fixed waste portion of each tank will use direct and in situ beta and gamma count measurements from the tank walls of each tank. The methodology described in this paper is subject to change due to processing changes and/or equipment availability, and performance during data collection.

The following sections discuss the estimation methodology. Section 2 presents the fixed and mobile waste component model for each tank. Sections 3 presents the estimation methodologies for the mobile waste components of each tank. Section 4 presents the estimation methodologies for the fixed waste components of each tank. Section 5 gives a summary of the methodology.
2.0 Model of Radionuclide Inventory

In order to develop a methodology for estimating the radionuclide inventory in Tanks 8D-1 and 8D-2, it is necessary to have a model that describes the radionuclide inventory for closure as Class C low-level waste (LLW) under 10 CFR 61.55. The tank inventories can currently be modeled as having two components of radionuclide waste, a mobile waste component and a fixed waste component. The mobile waste component, $I_{\text{Mobile}}$, consists of the radioactive liquid/slurry in each tank that is being pumped into the CFMT for melter feed preparation. The fixed waste component, $I_{\text{Fixed}}$, consists of the radionuclide waste inside the tank that adheres to the tank structures such as the wall, bottom, risers, heat exchanger, pumps, and gridwork that can not be pumped from the tanks.

Thus, at any time, the total radionuclide inventory in Tanks 8D-1 or 8D-2, $I_{\text{Total}}$, can be represented as:

$$I_{\text{Total}} = I_{\text{Mobile}} + I_{\text{Fixed}}$$  \hspace{1cm} (1)

where $I_{\text{Total}}$, $I_{\text{Mobile}}$, and $I_{\text{Fixed}}$ are vectors, with each component representing a specific radionuclide inventory in curies (Ci). Under 10 CFR 61.55 closure requirements, specific radionuclide inventories within the tanks must be statistically tested to meet specified threshold limits.

In reality, the waste is not neatly partitioned into mobile and fixed components the same way in both tanks. The problem arises in the mobile components of the waste. The mobile components of waste in each tank are characteristically different and are thus estimated differently in each tank. The mobile constituents of waste in Tank 8D-2 are generally homogeneous; in large part due to the size reduction of the zeolite as it is transferred from 8D-1 to 8D-2, reducing the zeolite particle size to approximately the same size as the sludge particles. This waste is assumed to be transferred at near 100% efficiency to the CFMT. This means that the transferred mobile waste in the CFMT from Tank 8D-2 is proportionally representative of the inventory remaining in 8D-2. Tank 8D-1; however, is a nonhomogeneous mix of supernatant, zeolite, and a small amount of sludge. The transfer efficiency from Tank 8D-1 to Tank 8D-2 is not 100% efficient and the transferred waste is not proportionally representative of the remaining waste in Tank 8D-1 and thus needs to be estimated differently.

The model assumes that data collected to estimate the mobile component of waste in each tank was collected while transfers occurred directionally from Tank 8D-1 to Tank 8D-2 and from Tank 8D-2 to the CFMT as presented in Figure 2.1. Variations on the flow of the mobile wastes, including transfers from Tank 8D-2 to Tank 8D-1, are not covered in this paper.
Figure 2.1. Assumed Flow of Mobile Waste Component of Waste.

The remainder of this paper discusses the estimation methodology that will be used for each tank to estimate the radiological components.
3.0 Tank 8D-1 and 8D-2 Mobile Inventory

The inventory estimation methodology for the mobile portion of Tank 8D-2 provides estimates, variances, and covariances for the total curies of 15 radiological constituents. At present, 6 of the 15 radionuclides are directly measured. The remaining 9 radionuclides are estimated by ratioing to various measured constituents. In the future, it is assumed that all except two of the radionuclides of interest will be directly measured due to an expansion of measurement techniques in the WVDP Analytical Laboratory.

The 8D-2 mobile inventory is estimated using data from CFMT concentration measurements. Radionuclide-containing waste is transferred directly from Tank 8D-2 to the CFMT. The CFMT is used to make up a batch of waste for one vitrification run in the glass melter. To do this, repeated transfers are made from Tank 8D-2 to the CFMT. In between transfers, the volume of the waste in both tanks may be altered by the addition (or removal) of water. Water is typically added to the feed in Tank 8D-2 to allow it to be mixed adequately, while water is typically removed from the CFMT to concentrate the waste.

The tank volumes are measured before and after each transfer and after any water additions or removals. During a transfer series, it is assumed the mobile material in Tank 8D-2 is homogeneous. It will also be assumed that the only thing added to Tank 8D-2 is water or transfers from Tank 8D-1.

The 8D-1 mobile inventory will also be derived partially from CFMT concentration measurements. The methodology for estimating the Tank 8D-1 inventory will entail keeping Tank 8D-2 in a steady state for several transfers to the CFMT both before and after the final transfer from 8D-1 to 8D-2. The Tank 8D-1 mobile waste inventory estimate will be calculated by first taking the difference between two Tank 8D-2 inventory estimates; the Tank 8D-2 inventory right after transfers from 8D-1 minus the 8D-2 “relative” heel inventory directly before 8D-1 transfers. From the difference between these values, an estimate of the ratio of the quantity of each radionuclide transferred from 8D-1 to the mass of zeolite transferred from 8D-1 will be made. These ratios will then be multiplied by the quantity of the waste remaining in 8D-1 to obtain the total mobile waste inventory of radionuclides in 8D-1.

3.1 Tank 8D-2 Mobile Waste Inventory Estimation

The Tank 8D-2 mobile waste constituents are estimated using concentration measurements taken from the CFMT. It assumed that a minimum of 9 CFMT samples will be measured from both the “relative” heel of the CFMT and after the final 8D-2 transfer of a transfer series. Concentration measurements obtained either by direct laboratory assays or by “ratioing,” are needed for the CFMT samples. Associated with the CFMT concentration measurements will be CFMT density measurements, CFMT volume measurements, and the transfer volume history from 8D-2. Currently, the directly measured radionuclides in the CFMT are Sr-90, Tc-99,
Cs-137, Pu-238, Pu-239+240, and Am241; while the ratio-derived radionuclides are Ni-63, I-129, Np-237, Pu-241, Pu-242, Cm-242, Cm 243+244, and Am-243. Figure 3.1 depicts the planned Tank 8D-2 transfers and sampling scheme for the Tank 8D-2 mobile estimation methodology.

Figure 3.1. CFMT Sampling and Tank 8D-2 Transfer Plan for Tank 8D-2 Mobile Inventory Estimation.

The assumptions in using the CFMT measurements to estimate the remaining mobile waste inventory and the associated uncertainty in 8D-2 are:

1. Both the CFMT and Tank 8D-2 mobile waste is homogenous and well-mixed.

2. In using “ratioing” to estimate the non-measured radionuclides, it is understood that the ratios are representative of the current contents of the tank.

3. There are no transfers from Tank 8D-1 to Tank 8D-2 during the mobile inventory measurement period.

4. There is no time lag effect in the CFMT concentration measurements when increases in the 8D-2 concentrations occur if a transfer is made from Tank 8D-1 before the inventory measurement period.

5. Volume, concentration, and density measurements have a quantifiable uncertainty.
Estimation occurs in two steps for the mobile waste inventory of Tank 8D-2. In the first step, an inventory of directly measured constituents is obtained for 8D-2. This direct inventory consists of the constituents directly measured in the CFMT during batch transfers. In the second step, this direct inventory is then completed by ratioing. Ratios between certain measured constituents and the unmeasured constituents have been computed using a radiochemical analysis of a historic sample from the Batch 10 vitrification run. This chemical analysis measured all desired constituents so that these ratios could be computed. The directly measured list of constituent estimates is then multiplied by these ratios to produce the missing constituents.

### 3.1.1 Direct Tank 8D-2 Mobile Inventory

To describe this procedure, the following variables are defined:

- **$V_{i1}$**: The volume of waste in 8D-2 before transfer $i$ is made.
- **$V_{i2}$**: The volume of waste transferred from 8D-2 to the CFMT during the $i$th transfer.
- **$V_{31}$**: The volume of waste in the relative “heel” of the CFMT.
- **$V_{32}$**: The volume of waste in the CFMT after the last transfer.
- **$I_i$**: The inventory in 8D-2 before the $i$th transfer is made. This is a vector, with each component describing the weight amount of a certain constituent in the tank.

Thus $I_1$ is the inventory in the tank at the start and, if $N$ transfers are made, $I_{N+1}$ represents the final inventory, the inventory we are interested in estimating. The amount of inventory in the tank after the $i$th transfer is given by $I_{i+1}$.

- **$C_1$**: The density adjusted concentrations (in weight/vol.) of the constituents in the relative “heel” of the CFMT. Only a partial set of constituent measurements is regularly taken.
- **$C_2$**: The density adjusted concentrations (in weight/vol.) of the constituents after the last transfer in the CFMT. Only a partial set of constituent measurements is regularly taken.
- **$P_i$**: The proportion of inventory transferred to the CFMT during transfer $i$. Assuming that 8D-2 is well mixed, this proportion must be:

$$P_i = \frac{V_{2i}}{V_{i1}} \quad (2)$$

A schematic of this mobile inventory methodology is given in Figure 3.2 depicting what the variables represent.

---

3.3
Given the above information, it is easy to establish a relationship between $I_i$ and $I_{i+1}$. It is:

$$I_{i+1} = (1 - P_i)I_i$$

and if we combine these, we find the final inventory is:

$$I_{N+1} = \prod_{i=1}^{N}(1 - P_i)I_1 = QI_1$$

We can use this relationship to produce a formula that relates $I_{N+1}$ to the total inventory transferred to the CFMT, $I_1$. The formula is:

$$I_{N+1} = \prod_{i=1}^{N}(1 - P_i)I_1 = QI_1$$
\[ \Delta I = I_1 - I_{N+1} = \left( \frac{1}{Q} - 1 \right) I_{N+1} \]  

(5)

This is the basic formula we use to estimate \( I_{N+1} \). The volume measurements give us \( Q \), and the CFMT measurements give us \( \Delta I \), the inventory transferred out of 8D-2.

To complete the estimation scheme, the formula for \( \Delta I \) is derived:

\[ \Delta I = V_{32} C_2 - V_{31} C_1 \]  

(6)

so the formula for computing the direct inventory, \( I_{Direct} \), from available measurements can be summarized as:

\[ I_{Direct} = I_{N+1} \]

\[ = \frac{Q}{1 - Q} \left( V_{32} C_2 - V_{31} C_1 \right) \]

\[ = H(V, C) \]  

(7)

where \( H(V, C) \) is a function representing the above computation.

### 3.1.2 Using Ratios to Complete the Tank 8D-2 Inventory

The complete vector of constituents is produced by multiplying certain measured constituents, or functions of constituents, by ratios to produce the missing constituents. Suppose \( N \) constituents out of a total of \( M \) are measured. We want to construct a vector, \( I_{Mobile} \), in which the first \( N \) components represent the measured constituents and the remaining \( M-N \) represent the unmeasured. The computation can be described by the following form:

\[ I_{Mobile} = F(R, I_{Direct}) \]  

(8)

\( R \) represents a vector of ratios used to estimate the \( M-N \) constituents. The notation \( F(R, I_{Direct}) \) emphasizes the fact that \( I_{Mobile} \) is a function of the ratios.

### 3.1.3 Propagation of Errors for Tank 8D-2 Inventory

Based on the formulas derived in the previous sections, the propagation of error formulas for \( \text{Cov}(I_{Direct}) \) and \( \text{Cov}(I_{Mobile}) \) can be written. They are:
\[
\text{Cov}(I_{\text{Direct}}) = \left( \frac{\partial H}{\partial V} \right)^T \text{Cov}(V) \left( \frac{\partial H}{\partial V} \right) + \left( \frac{\partial H}{\partial C} \right)^T \text{Cov}(C) \left( \frac{\partial H}{\partial C} \right)
\]

(9)

and

\[
\text{Cov}(I_{\text{Mobile}}) = \left( \frac{\partial F}{\partial R} \right)^T \text{Cov}(R) \left( \frac{\partial F}{\partial R} \right) + \left( \frac{\partial F}{\partial I_{\text{Direct}}} \right)^T \text{Cov}(I_{\text{Direct}}) \left( \frac{\partial F}{\partial I_{\text{Direct}}} \right).
\]

(10)

In order to calculate the uncertainties (i.e., the covariance matrix) of \( I_{\text{Direct}} \), we must assemble the following inputs:

\text{Cov}(V): The covariance matrix of all measured volumes. Because of the way batch processing is done and volume measurements are determined, these measurements are assumed to be independent. In this case, \( \text{Cov}(V) \) is a diagonal matrix.

\text{Cov}(C): The covariance matrix of the measured concentrations in the CFMT. The covariance matrix is built by pooling the CFMT concentration data, after adjusting for the mean in each transfer period. Because the measurements utilize a common density conversion, they will not be independent and \( \text{Cov}(C) \) will not be diagonal.

\text{Cov}(R): The uncertainties in the ratios. This matrix will also not be diagonal because the ratios contain common denominators.

### 3.2 Tank 8D-1 Mobile Waste Estimation Methodology

The Tank 8D-1 mobile waste inventory will be determined by transferring a portion of the 8D-1 waste to 8D-2 and then using the methodology in Section 3.1 to determine the inventory of the transfer. This will require performing the following in order:

1. Estimation of the inventory in the Tank 8D-2 “relative” heel.
2. Transfer of waste from 8D-1 to 8D-2.
3. Estimation of the Tank 8D-2 inventory after the 8D-1 transfers.

The transfer inventory is then estimated by the difference between the two Tank 8D-2 inventories.

If the waste transferred from 8D-1 to 8D-2 were homogeneous, then the transfer inventory could easily be used to determine the 8D-1 inventory by multiplying by volume ratios. However, the 8D-1 waste is not homogeneous. To produce an inventory estimate for Tank 8D-1, we will use
the assumption that the radionuclide waste should be mostly in the zeolite solids in the tank and that the radionuclide waste should be a fixed percentage of the zeolite. To use this strategy, a zeolite inventory estimate will be needed for the transferred inventory to 8D-2 and also for the remaining zeolite in Tank 8D-1. The Tank 8D-1 remaining inventory will then be estimated by multiplying the ratio of the radionuclide transfer inventory to the mass of transferred zeolite by the remaining mass of zeolite in Tank 8D-1.

Assumptions that are valid for estimating 8D-2 inventories are also applicable to 8D-1 inventory estimates since the difference between the pre-8D-1 and post-8D-1 transfer inventories depends on 8D-2. Additional assumptions in using the CFMT measurements to estimate the remaining waste inventory and the associated uncertainty in 8D-1 are:

1. The Class C radionuclides are generally in two components of the solids in 8D-1: the sludge and zeolite. At the time of the final 8D-1 transfers to 8D-2, the sludge phase transferred from Tank 8D-1 should account for a negligible increase in the Tank 8D-2 inventory.

2. The zeolite phase has a fixed, but unknown, composition, except for the marker species, which are Cs-137, Al, and Si.

3. The 8D-2 transfer to CFMT inventory estimation methodology can be used to measure the radionuclide/marker concentrations both before and after the final 8D-1 transfer.

4. The inventory or concentration of the zeolite in the residual 8D-1 waste can be estimated

3.2.1 Tank 8D-1 Inventory Estimation from CFMT Measurements and Zeolite Mass Estimates for Tank 8D-1

To describe the estimation procedure, the following vector variables are defined:

$I_{R1}$: The inventory of radionuclides in Tank 8D-2 before the final transfer from Tank 8D-1

$I_{R2}$: The inventory of radionuclides in Tank 8D-2 after the final transfer from Tank 8D-1

$I_{\text{Mobile}}$: The inventory of radionuclides in Tank 8D-1 after the final transfer from Tank 8D-1 to Tank 8D-2

and the following scalar variables:

$I_{Z1}$: The inventory of zeolite in Tank 8D-2 before the final transfer from Tank 8D-1

$I_{Z2}$: The inventory of zeolite in Tank 8D-2 after the final transfer from Tank 8D
CZ: The concentration of zeolite remaining in Tank 8D-1 after the final transfer to Tank 8D-2

V_{8D-1,i}: The total volume of mobile waste remaining in Tank 8D-1 after the final transfer to Tank 8D-2.

A schematic of this mobile inventory methodology for Tank 8D-1 is given in Figure 3.3 depicting what the variables represent.

Thus, for any particular radionuclide i in \( I_{\text{Mobile}} \), the estimated inventory in Tank 8D-1 of that radionuclide is given by:

\[
I_{\text{Mobile},i} = \frac{(I_{R2,i} - I_{R1,i})}{(I_{Z2} - I_{Z1})} CZV_{8D1} \tag{11}
\]

where \( I_{R2,i} - I_{R1,i} \) and \( I_{Z2} - I_{Z1} \) denote the transfer inventory of radionuclides and zeolite,
respectively. To simplify the notation, let \( A \) be the vector of the concatenated variables \( I_{R1}, I_{Z1}, I_{R2}, I_{Z2}, C_Z, \) and \( V_{8D-1} \), that is, \( A = (I_{R1}, I_{Z1}, I_{R2}, I_{Z2}, C_Z, V_{8D-1}) \). Then \( I_{Mobile} \) can be defined as a vector valued function \( G(A) \) where

\[
I_{Mobile} = G(A) = \begin{pmatrix}
I_{R2,1} - I_{R1,1} & C_Z V_{8D1} \\
I_{Z2} - I_{Z1} & . \\
. & . \\
. & . \\
I_{R2,1} - I_{R1,1} & C_Z V_{8D1}
\end{pmatrix}
\]  

(12)

3.2.2 Propagation of Errors for Tank 8D-1 Inventory

Based on the formulas derived in the previous sections, one can write down the propagation of error formulas for \( Cov(I_{Mobile}) \) as

\[
Cov(I_{8D1}) = \left( \begin{array}{c}
\frac{\partial G}{\partial A}
\end{array} \right)^T Cov(A) \left( \begin{array}{c}
\frac{\partial G}{\partial A}
\end{array} \right).
\]

(13)

In order to calculate the covariance matrix of \( I_{Mobile} \), we must assemble the following inputs since the \( Cov(A) \) is composed of several submatrices.

\( Cov(I_{R1,I_{Z1}}) \) and \( Cov(I_{R2,I_{Z2}}) \): The covariance matrix of the concatenated vectors \( (I_{R1}, I_{Z1}) \) and \( (I_{R2}, I_{Z2}) \) is obtained from the methodology defined for \( I_{Mobile} \) as discussed in Section 3.1.

\( Var(C_Z) \): The variance of the concentration of zeolite remaining in Tank 8D-1. This value will be determined from direct estimation, historical knowledge, or expert opinion.

\( Var(V_{8D-1}) \): The variance of the volume of the total mobile waste remaining in Tank 8D-1 can be obtained from tank readings.

It will be assumed that \( (I_{R1}, I_{Z1}), (I_{R2}, I_{Z2}), C_Z, \) and \( V_{8D-1} \) are independent.
4.0 Tank 8D-1 and 8D-2 Fixed Waste Inventory

The fixed waste portion of the total radionuclide inventory in Tanks 8D-1 and 8D-2 are defined as the inventory adhering to the tank walls, bottom, top, risers, and gridwork that is not part of the mobile waste. The fixed waste profile will be modeled in each tank as a stratified design defined on the internal surface area of the tank.

The fixed waste in the tanks will be measured using an in situ beta/gamma detector to scan the walls of each tank from approximately 5 feet from the top of the tank down to the top of the remaining mobile waste. Three scans of the tank walls will be made in different locations, with count data collected approximately every 6 inches. The count data will be translated into curies/meter$^2$ concentrations of Sr-90 and Cs-137. These concentrations will then be scaled using ratios developed from direct samples taken from the tank walls either from a burnishing sampler or from swab surface samples.

These direct samples will be collocated with areas from the beta/gamma detector scans of the walls. At least three samples will be taken from each identified strata on the tank walls. These samples will then be analyzed in the WVDP Analytical Laboratory for radionuclide concentrations. Ratios, (i.e., scaling factors based on analytic data) will be then be developed from these samples to adjust the beta/gamma counts of Sr-90 and Cs-137 to estimate the total amount of the radionuclides in the fixed waste component of waste.

Assumptions in developing estimates of the fixed waste inventory in 8D-2 are:

1. The beta/gamma detector obtains a representative sample of the tank walls.
2. The internal tank structures (columns, heat-exchanger, etc.) have the same fixed waste profile as the walls.
3. Ratios for translating the gross beta and gamma counts to estimates for all radionuclides are representative and in the future will be developed from the actual tank wall samples.

4.1 Tank 8D-1 and 8D-2 Fixed Waste Estimation Methodology

First, the stratification of the fixed waste within the tanks will be performed separately. For each tank, the stratification will be accomplished by using a hierarchical statistical clustering algorithm using the beta/gamma count data. The variables for stratification will be: height from the bottom of the tank, and the curies of Sr-90 and Cs-137 per square meter. Once the stratification is complete, the estimation procedure can be implemented.
To describe the estimation procedure, the following vector variables are defined:

\( R_h \): The average of the ratios of the Class C radionuclides (in curies) to the Cs-137 or Sr-90 counts developed from the collocated samples within strata \( h \)

\( C_{hi} \): The Cs-137 and Sr-90 concentrations for observation \( i \) in strata \( h \)

and the following scalar:

\( A_h \): Total area of the interior of Tank 8D-2 in strata \( h \) in squared units of measurement.

Then the fixed waste inventory in Tank 8D-2 or 8D-1 can be estimated as the sum of scalars times the Hadamard product of the two vectors: \( R_h \) and \( \bar{C}_h \):

\[
I_{\text{Fixed}} = \sum_h A_h R_h \ast \bar{C}_h = \sum_h A_h K(R_h, \bar{C}_h) \tag{14}
\]

where

\[
\bar{C}_h = \sum_i \frac{C_{hi}}{n_h} \tag{15}
\]

and \( n_h \) is the number of beta/gamma count observations in strata \( h \). The notation \( K(\bar{C}_h, R_h) \) emphasizes that \( I_{\text{Fixed}} \) is a function of the ratios and counts.

### 4.2 Propagation of Errors for Tank 8D-1 and Tank 8D-2 Fixed Inventory

Based on the formulas derived in the previous sections, the propagation of error formulas for fixed waste component of each tank can be written as

\[
\text{Cov}(I_{\text{Fixed}}) = \sum_h A_h^2 \left( \frac{\partial K}{\partial \bar{C}_h} \text{Cov}(\bar{C}_h) \frac{\partial K}{\partial R_h} + \frac{\partial K}{\partial R_h} \text{Cov}(R_h) \frac{\partial K}{\partial R_h} \right) \tag{16}
\]

where:

\( \text{Cov}(\bar{C}_h) \): The covariance matrix of the average beta/gamma counts in strata \( h \). This will not be a diagonal matrix.

\( \text{Cov}(R_h) \): The covariance matrix of the ratios of the directly measured radionuclides and the collocated count data. This matrix will not be diagonal as the ratios have
common denominators of either Cs or Sr.
5.0 Summary

The total inventory of radionuclide waste in both Tanks 8D-1 and 8D-2 is simply the sum of the mobile and fixed waste components of each tank. The covariance matrix of each inventory is given as the sum of the covariance term of each component: Cov(I_{Mobile}) + Cov(I_{Fixed}) as it will be assumed that I_{Mobile} and I_{Fixed} are independent for purposes of estimation.

In conclusion, the utility of the methodology depends on:

1. The validity of the assumptions in the estimation procedures.
2. The ability to obtain estimates of the zeolite concentration in Tank 8D-1 with an associated uncertainty estimate.
3. The continued operation of the CFMT sampler system.
4. The ability of the WVDP Analytical Laboratory to continue to provide reliable measurement data.

Improvements in the estimation procedure can be obtained if less ratio estimation is performed in estimating Tank 8D-2 inventories. This can be accomplished by measuring as many radionuclides of concern in the WVDP Analytical Laboratory as possible. This would alleviate the dependence on using the Batch 10 data to develop ratios and eliminate a component of uncertainty in the estimation procedure. Also, reduction in the error terms can be achieved if more CFMT concentrations are obtained. However, the number of samples beyond the nine already planned should be carefully assessed so as not to confound statistically significant reduction in the error terms from practical and real scientific reduction in the error terms.
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