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Hector V 3.5.0

Equations & Parameters

February 2026

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1 Introduction

Originally introduced by Hartin et al. [2015], Hector is a community Reduced Complexity Model (RCM) designed to simulate changes in Earth’s energy balance and carbon cycle. It is maintained and developed by the Pacific Northwest National Laboratory. Hector has continued to evolve with the advances in science, leading to different releases over the years. The latest version documented in a peer-reviewed publication is Hector V3.2.0, as described by Dorheim et al. [2024].

This document corresponds to the V3.5.0 release (see <https://github.com/JGCRI/hector/releases>) and serves as an update rather than an exhaustive documentation manuscript. Building on Dorheim et al. [2024], it focuses on the modifications in equations and parameters introduced since the V3.2.0 release.

1.1 Notation

The common notation used throughout this document includes the following listed in Table 1.1. Hector variable names will be consistent across sections and equations; however, parameter symbols may be reused. See the parameter table for each section for the relevant parameter description.

Notation	Description
E_X	Global annual emissions of X from input
$[X]$	Global Atmospheric concentration of X
X_0	The subscript 0 appended to any variable indicates a preindustrial value
RF_X	Hector uses effective radiative forcing (ERF, [Smith et al., 2021]) as the driver for changes in atmospheric energy balance. In general, where “radiative forcing” is mentioned, this refers to ERF. A number of the parameterizations for species-specific forcing used in hector were originally in terms of SARF. Where necessary, these parameterizations are adjusted with a multiplicative factor to produce effective radiative forcing. We calibrate ERF changes over the historical period largely to AR6 values, or, in a few cases, using subsequent literature where updates are available.
$SARF_X$	Stratospheric-temperature-adjusted radiative forcing X
nat.	Refers to natural, typically in the context of naturally occurring emissions
strat.	Stratosphere
trop.	Troposphere
t	Simulation year

Table 1: Common symbols and notation used consistently throughout this document.

2 CH₄

The V3.5 changes to Hector’s CH₄ include: enabling time varying natural CH₄ emissions ($E_{\text{nat. } CH_4}(t)$) during the historical period; new coefficients for the equation used to calculate the CH₄ chemical lifetime due to the reaction with hydroxyl radicals (τ_{OH}); and new parameterizations for CH₄’s soil and stratospheric sinks.

$$\Delta[CH_4](t) = \frac{E_{CH_4}(t) + E_{\text{nat. } CH_4}(t) + Rh_{CH_4}(t)}{2.78} - \frac{[CH_4](t-1)}{\tau_{OH}(t)} - \frac{[CH_4](t-1)}{\tau_{\text{strat.}}} - \frac{[CH_4](t-1)}{\tau_{\text{soil}}} \quad (1)$$

$$a(t) = c_{CH_4} \ln \left(\frac{[CH_4](t)}{[CH_4]_0} \right) \quad (2a)$$

$$b(t) = c_{NO_x} (E_{NO_x}(t) - E_{NO_x0}) \quad (2b)$$

$$c(t) = c_{CO} (E_{CO}(t) - E_{CO0}) \quad (2c)$$

$$d(t) = c_{NMVOC} (E_{NMVOC}(t) - E_{NMVOC0}) \quad (2d)$$

$$(2e)$$

$$\tau_{OH}(t) = \tau_{OH0} \times e^{-(a(t)+b(t)+c(t)+d(t))} \quad (3)$$

$$[CH_4](t) = [CH_4](t-1) + \Delta[CH_4](t) \quad (4)$$

$$SARF_{CH_4}(t) = \left(a_3 \sqrt{[CH_4](t-1)} + b_3 \sqrt{[N_2O](t-1)} + d_3 \right) \left(\sqrt{[CH_4](t-1)} - \sqrt{[CH_4]_0} \right) \quad (5)$$

$$RF_{CH_4}(t) = (\delta_{CH_4} + 1) SARF_{CH_4}(t) \quad (6)$$

Name	Description
[CH ₄]	atmospheric CH ₄ concentration Eq. 4, ppbv
[N ₂ O]	atmospheric N ₂ O concentration Dorheim et al. [2024] Eq. S1 , ppbv
$E_{\text{nat. } CH_4}$	natural CH ₄ emissions § 2.1, Tg
τ_{OH}	CH ₄ sink or lifetime due to OH Eq. 3, years
Rh_{CH_4}	CH ₄ from heterotrophic respiration of permafrost [Woodard et al., 2021], Tg
2.78	conversion factor from emissions to concentrations of CH ₄ [Wigley et al., 2002]

Table 2: Variables and notation used in Equations 1 - 6 related to calculating CH₄ concentrations and radiative forcing.

2.1 Natural CH₄ Emissions

Natural CH₄ emissions vary during the historical period before and remain constant after 2015. The historical natural CH₄ emissions are calculated by inverting Equation 1 while using Meinshausen et al. [2020] CH₄ concentrations and Rh_{CH_4} Hector results from a Hector simulation where historical CH₄ concentrations are prescribed. Beyond 2015, natural CH₄ emissions are held constant at the 2005-2015 mean.

The net effect of adding variable natural historical CH₄ emissions is to make Hector’s historical concentration values match observations, which is now the default set-up. We note that factors other than natural emissions will impact historical concentrations, including any biases in the parameterizations used for methane lifetime calculations and/or biases in the historical anthropogenic emission time series. The parameterizations used in Hector do not account for short-timescale variations, but are captured by the time varying natural emissions so that the resulting historical concentration time series matches observations.

Symbol	Description	Value	Units	Notes
$\tau_{\text{strat.}}$	stratospheric CH_4 sink lifetime	150	years	[IPCC, 2014]
τ_{soil}	CH_4 soil sink lifetime	120	years	[IPCC, 2014]
c_{CH_4}	Eq. 3 parameter	-0.32		[Wigley et al., 2002]
c_{NO_x}	Eq. 3 parameter	8.4e-3		§ 2.2
c_{CO}	Eq. 3 parameter	-1.575e-4		§ 2.2
c_{NMVOC}	Eq. 3 parameter	-4.725e-4		§ 2.2
τ_{OH_0}	preindustrial OH sink lifetime	9.6	years	[Wigley et al., 2002]
a_3	Eq. 5 parameter	-8.9603e-5		7.SM.1 of [Smith et al., 2021]
b_3	Eq. 5 parameter	-1.2462e-4		7.SM.1 of [Smith et al., 2021]
d_3	Eq. 5 parameter	0.045194		7.SM.1 of [Smith et al., 2021]
δ_{CH_4}	tropospheric adjustment parameter	-0.14	(unitless)	7.SM.1.3.1 of [Smith et al., 2021]

Table 3: Parameter values, description, and sources used in equations equations 1, 4, 3, 5 related to calculating CH_4 concentrations and radiative forcing.

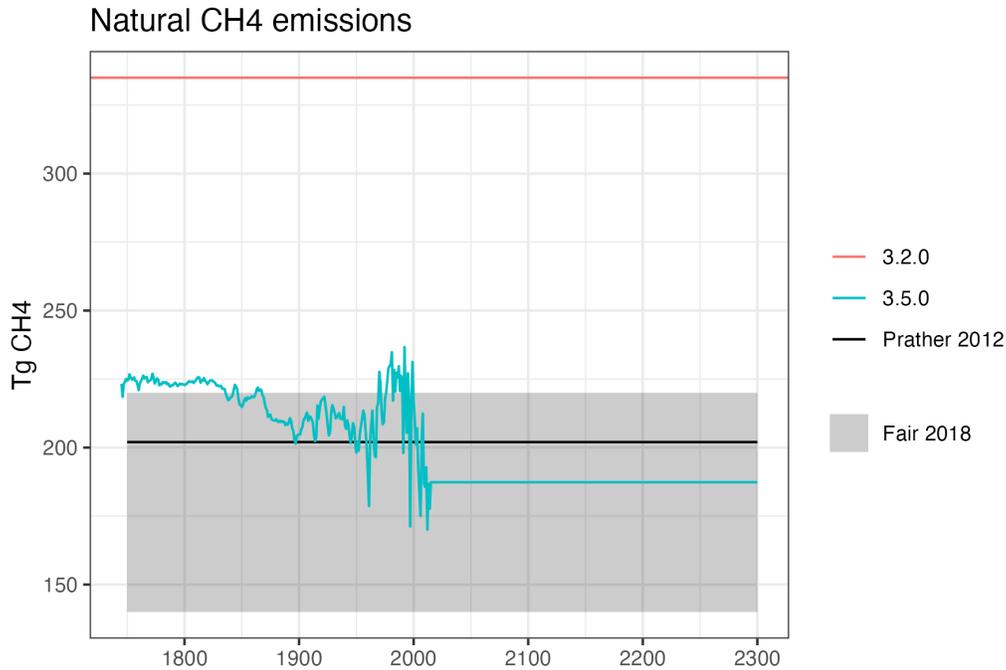


Figure 1: Comparison of the natural CH_4 emissions used by Hector v3.2.0 and v3.5.0 with Prather et al. [2012] and Smith et al. [2018].

2.2 Deriving c_{NO_x} , c_{CO} , and c_{NMVOC}

Since Hector V1 Hartin et al. [2015], CH_4 chemical lifetime due to the reaction with hydroxyl radicals (τ_{OH}) has been modeled using an approach taken from Wigley et al. [2002] from OxComp Workshop results [IPCC, 2001]. While the same function continues to be used today, see equation (3), advances in science and subsequent Earth System model development indicate that Hector has been underestimating the effects that NO_x , CO , and $NMVOC$ have on τ_{OH} .

Thornhill et al. [2021] evaluated results from the CMIP6 AerChemMIP project to quantify the impact of methane, NO_x , and VOCs on methane lifetime. While there was some variability between models, the largest impact was from NO_x emissions, followed by methane’s impact on its own lifetime, with a generally somewhat smaller impact from VOCs (although inter-model variability for VOCs was particularly large).

We evaluated the methane lifetime parameterization originally used in Hector (Equation S6 of Dorheim et al. [2024]) on methane lifetime between 1850 and 2014 using the same historical emissions used in CMIP6. We found that, while methane’s impact on its own lifetime was similar to Thornhill et al. [2021], the impact of NO_x was underestimated by a factor of 2 and that of VOCs by 50%. While the impact of CO was not separately evaluated in CMIP6, we assume the same change to the CO coefficient as that inferred for VOCs given the similar role that CO plays to VOCs.

Before these changes, the methane lifetime in Hector increased substantially over the historical period. With these modifications, the methane lifetime begins to decrease after about 1960 (due to increasing NOx emissions), which better matches the overall time trend of methane lifetime inferred from CMIP6 models from Stevenson et al. [2020] and Smith et al. [2024].¹

Parameter	Wigley et al. [2002] Value	α Adjustment scalar	New Value
c_{NO_x}	0.0042	2	8.4e-3
c_{CO}	-0.000105	1.5	-1.575e-4
c_{NMVOC}	-0.000315	1.5	-4.725e-4

Table 4: Parameters used in equations (2) and (3).

3 N_2O

The only change in Hector V3.5 related to N_2O is that now historical natural N_2O emissions vary over time before being held constant starting in 2015.

$$\Delta[N_2O](t) = \frac{E_{N_2O}(t) + E_{nat. N_2O}(t)}{4.8} - \frac{[N_2O](t-1)}{\tau_{N_2O}(t-1)} \quad (7)$$

$$[N_2O](t) = [N_2O](t-1) + \Delta[N_2O](t) \quad (8)$$

$$\tau_{N_2O}(t) = \tau_0 \times \left(\frac{[N_2O](t)}{[N_2O]_0} \right)^{-0.05} \quad (9)$$

$$SARF_{N_2O}(t) = \left(a_2 \sqrt{[CO_2](t)} + b_2 \sqrt{[N_2O](t)} + c_2 \sqrt{[CH_4](t)} + d_2 \right) \left(\sqrt{[N_2O](t)} - \sqrt{[N_2O]_0} \right) \quad (10)$$

$$RF_{N_2O}(t) = \delta_{N_2O} SARF_{N_2O}(t) + SARF_{N_2O}(t) \quad (11)$$

¹We appreciate conversations with Vaishali Naik regarding literature results for methane lifetime.

Name	Description
$[N_2O]$	Atmospheric N_2O concentration Eq. 8, ppbv
E_{N_2O}	Anthropogenic N_2O emissions, Input, Tg N
$E_{nat.CH_4}$	natural N_2O emissions § 3.1, Tg
τ_{OH}	N_2O sink 9, years
$SARF_{N_2O}$	stratospheric-temperature-adjusted radiative forcing for N_2O Eq. 10, $W m^{-2}$
RF_{N_2O}	effective radiative forcing for N_2O Eq. 11, $W m^{-2}$

Table 5: Variables and notation used in Equations 7 - 11 related to calculating N_2O concentrations and radiative forcing.

Symbol	Description	Value	Units	Notes
$[N_2O]_0$	Preindustrial atmospheric N_2O	273.87	ppbv	[Smith et al., 2021]
τ_0	Preindustrial N_2O lifetime	132	yrs	[Hartin et al., 2015]
a_2	Eq. 10 parameter	-3.4197×10^{-4}	$W m^{-2} ppm^{-1}$	Table 7.SM.1 Smith et al. [2021]
b_2	Eq. 10 parameter	2.5455×10^{-4}	$W m^{-2} ppb^{-1}$	Table 7.SM.1 Smith et al. [2021]
c_2	Eq. 10 parameter	-2.4357×10^{-4}	$W m^{-2} ppb^{-1}$	Table 7.SM.1 Smith et al. [2021]
d_2	Eq. 10 parameter	0.12173	$W m^{-2} ppb^{-1/2}$	Table 7.SM.1 Smith et al. [2021]
δ_{N_2O}	tropospheric adjustment parameter	0.07	unitless	7.SM.1.3.1 of Smith et al. [2021]

Table 6: Parameters and variables related to calculating the change in N_2O concentrations (Eq. 7), stratospherically adjusted halocarbon radiative forcing (Eq. 10), and radiative forcing (Eq. 11), where t indicates the simulation year.

3.1 Natural N_2O Emissions

Natural N_2O emissions vary during the historical period before and remain constant after 2015. The historical natural N_2O emission are calculated by inverting Equation 7 while using Meinshausen et al. [2020] of the N_2O concentrations. Beyond 2015, natural N_2O emissions are held constant at the 2005-2015 mean. Previously natural N_2O emissions were held constant.

The net effect of adding variable natural historical N_2O emissions is to make Hector’s historical concentration values match observations, which is now the default set-up. We note that factors other than natural emissions will impact historical concentrations, including any biases in the parameterizations used for N_2O lifetime calculations and/or biases in the historical anthropogenic emission time series. The parameterizations used in Hector do not account for short-timescale variations, but are captured by the time varying natural emissions so that the resulting historical concentration time series matches observations.

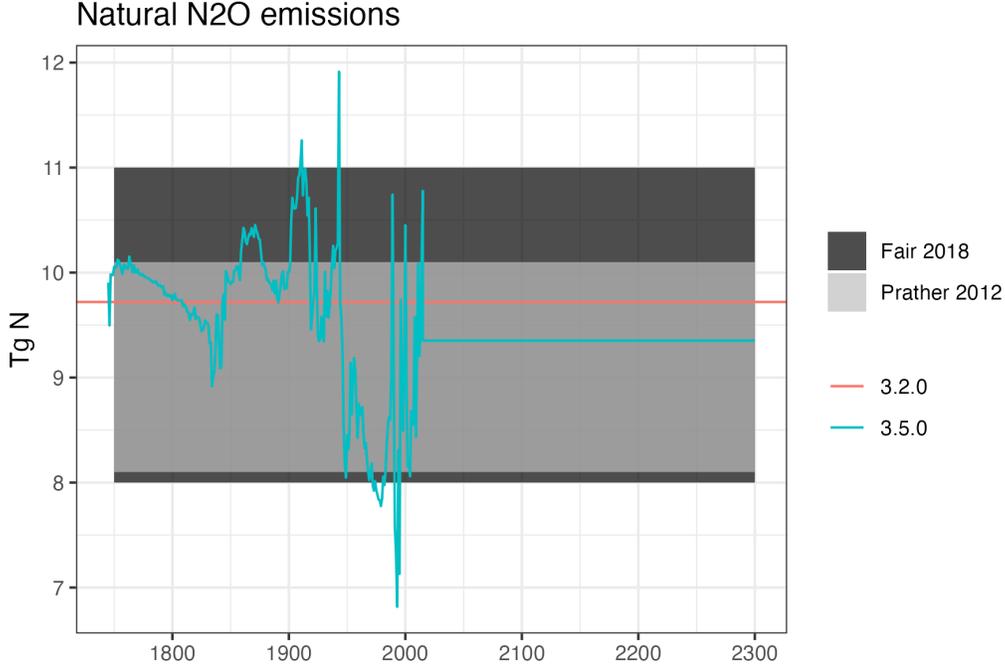


Figure 2: Comparison of the natural N₂O emissions used by Hector v3.2.0 and v3.5.0 with Prather et al. [2012] and Smith et al. [2018].

4 Calibrating to Observations

Before each major Hector release, we calibrate a handful of Hector parameters to historical observations, so that Hector global average temperature, CO₂ concentrations, and ocean heat content are consistent with observations.

As in Dorheim et al. [2024], the vertical ocean heat diffusivity (κ), CO₂ fertilization factor (β), and heterotrophic respiration temperature sensitivity factor (Q_{10}) were treated as free parameters. We found the default parameter values using the Nelder–Mead optimization routine Nelder and Mead [1965] to minimize the average mean squared error between Hector results and observations. As in Dorheim et al. [2024] Hector’s global mean surface temperature from 1850 - 2015 were compared with Morice et al. [2021] and Hector’s CO₂ concentrations from 1850 - 2015 were compared with Meinshausen et al. [2020]. However, for calibrating Hector V3.5.0 we also included 1957 - 2015 ocean heat content from Kuhlbrodt et al. [2023] as an observational constraint.

Parameter	Value
κ	1.042
β	0.65
Q_{10}	1.2

Table 7: Fitted parameter values.

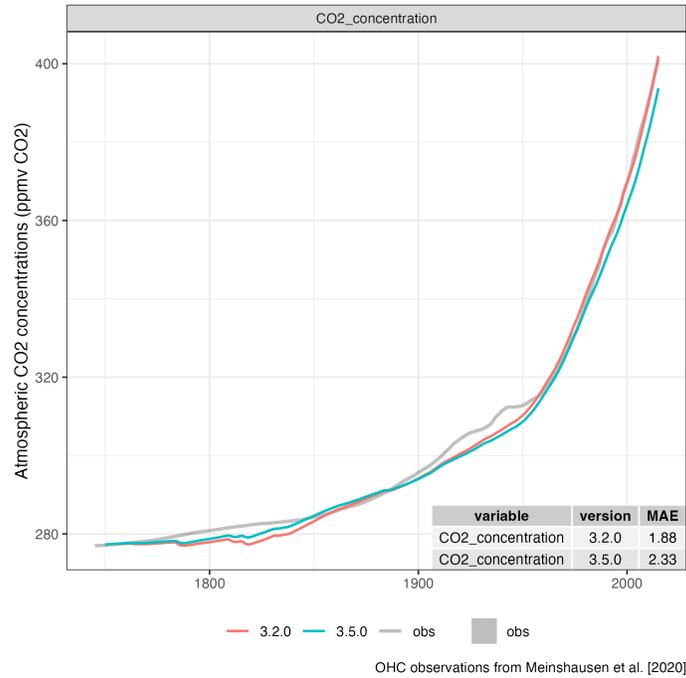


Figure 3: Comparison between Hecator V 3.5.0 and V 3.2.0 with the Meinshausen et al. [2020] CO₂ concentrations. The MAE quantifies the error between the Hecator results and the observations.

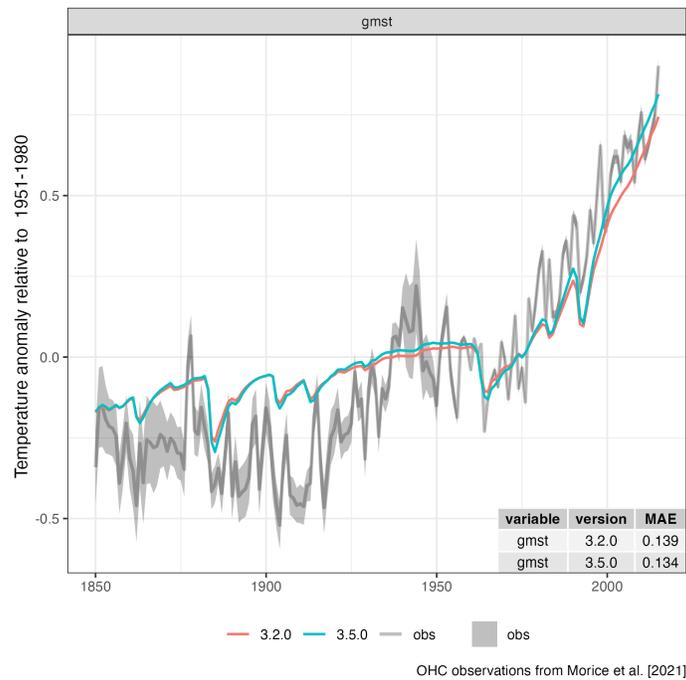


Figure 4: Comparison between Hecator V 3.5.0 and V 3.2.0 with the Morice et al. [2021] mean global surface temperature anomaly. The MAE quantifies the error between the Hecator results and the observations.

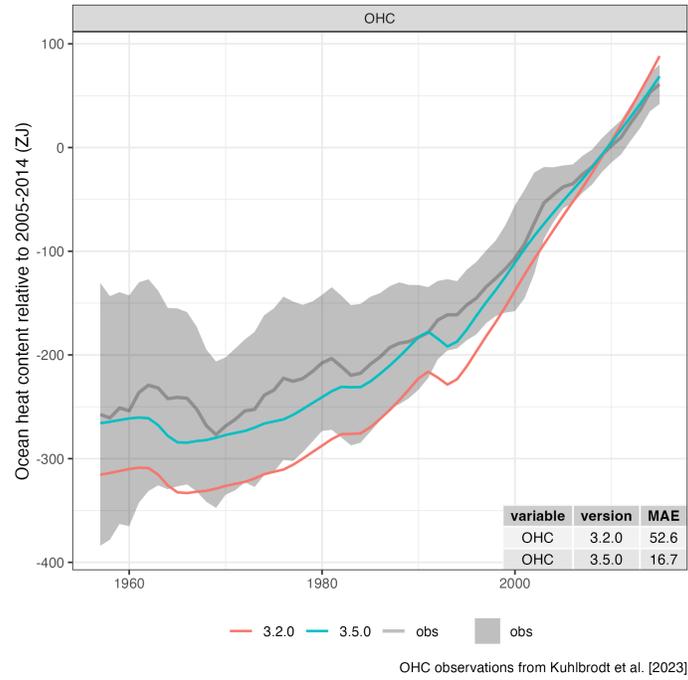


Figure 5: Comparison between Hektor V 3.5.0 and V 3.2.0 with the Kuhlbrodt et al. [2023] ocean heat content. The MAE quantifies the error between the Hektor results and the observations.

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