

# Model Formulations

Integrating Distributed Energy  
Resources (DER) using Advanced Unit  
Commitment Models and DER  
Aggregation Methodologies

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## **Abstract**

A distribution energy resource aggregator (DERA) constitutes a group of distribution energy resources with small generation capacities that, when combined, meet the threshold to participate in the electricity wholesale market. This document investigates the DERA offer model formulation with three components of solar, battery energy storage system (BESS), and the demand response resources (DRRs). Different economical assessment methodologies have been developed to incorporate bids for individual distributed resources, including a solar offer model, BESS opportunity cost offer algorithm, and nested utility function model for DRRs. In addition, an optimization-based method and a cost-based method are proposed to aggregate individual cost offers to a DERA cost curve to bid in the wholesale electricity market while the schedule-following method and profit-following method are also introduced to simulate the DER actual dispatch. Based on the DERA models in this document, the SCUC-DER project will be able to assess the impacts of DERA on the distribution system's operation and reliability.

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# 1 Overview

FERC Order 2222 has significantly impacted distributed energy resources (DER) participation rules in US electricity markets. The order lowers the power threshold to 0.1 MW to remove barriers and enable broader participation of DERs, such as solar panels, batteries, and electric vehicles, in wholesale electricity markets. It requires grid operators to revise their market rules to accommodate DER aggregations, allowing them to participate alongside traditional generators (Eldridge and Somani, 2022).

While grid operators are required to revise their market rules to accommodate distribution energy resources aggregations (DERAs), there are a lot of challenges to be addressed in registration, bidding, scheduling, and settlement processes to ensure fair and efficient participation (ERCOT, 2020; ISO-NE, MISO, 2022; NYISO, PJM, 2021). However, DER aggregation is still in early stages of development (Lu *et al.*, 2019; Wang *et al.*, 2019; Ross and Mathieu, 2021) because it is difficult to find quality data on how DER assets are economically valued and aggregated (Alshehri *et al.*, 2020; Su and Kirschen, 2009). This document presents a general model to better value each distributed resource both economically and computationally efficiently.

The DERA model considered here consists of three components: solar, battery energy storage system (BESS) and demand response resource (DRR). Hybrid mix of solar with battery backup is widely installed for residential and commercial applications while DRR can be employed in nearly any distribution feeder and is generally expected to be an important component for future grid operations. In general, each DER aggregator maximizes its profit through optimized bidding strategies in the wholesale market, which includes the optimal charging/discharging schedules for BESS and best price-responsive actions for DRR.

This document first demonstrates detailed offer models for each individual resource. For the DRRs, the associated money-valued cost for demand response actions is derived using generic economic consumption on a nested utility function. By treating energy consumption at each of arbitrary time periods as a distinct good, the time preference of energy consumers is decoded to represent its response to price changes over time. The detailed steps are shown about how to generalize the utility function with different choices of parameters, such as elasticity parameters, share parameters, etc. For the BESS, an opportunity cost-based offer is incorporated because other typical costs like degradation costs and fixed costs requires binary or nonlinear constraints. This paper focuses on the deriving the price-quantity cost curves for charging and discharging based on the energy price estimates in the future. Three approaches are proposed for computing opportunity cost curves: an approximation method, which estimates costs over a backward and forward time window; an optimization method, which calculates costs at each period by fixing quantities for that time interval in the profit-maximization problem; and a deterministic method, which identifies the closest upcoming profitable opportunity for the battery.

With cost offers derived for each resource type above, the aggregation offer blocks are constructed for further bidding of DERA into the wholesale market. Two methods are demonstrated to aggregate the individual costs based on DERA's baseline dispatch schedule. The optimization-based method generates the offer curves by optimizing the dispatch of resources at each time step while the cost-based method directly merges cost curves of each resource type into blocks.

Finally, the optimal dispatch schedule is disaggregated for each resource in DERA with schedule following method or profit following method. The former obtains the device-level dispatches at each time  $t$  by solving the deviation minimization problem from the dispatch baseline while the latter by maximizing DERA's total profit.

## 1.1 Nomenclature

### 1.1.1 Sets and Indices

#### Sets

$i, j \in \mathcal{I}$	Buses
$k \in \mathcal{K}$	All distributed energy resources (DERs)
$l \in \mathcal{L}$	Transmission lines
$n \in \mathcal{N}$	DER aggregators
$o \in \mathcal{O}$	Blocks in DERA offer
$t \in \mathcal{T}$	Time intervals in the market clearing horizon

#### Subsets

$\mathcal{K}_i^{\text{bus}} \subset \mathcal{K}$	Set of DER resources connected to bus $i$ .
$\mathcal{K}_n \subset \mathcal{N}$	Set of all Individual DERs in DERA $n$
$\mathcal{K}_n^{\text{BAT}} \subset \mathcal{K}_n$	Set of battery storage resources in DERA $n$
$\mathcal{K}_n^{\text{DRR}} \subset \mathcal{K}_n$	Set of demand response resources (DRRs) in DERA $n$
$\mathcal{K}_n^{\text{SOL}} \subset \mathcal{K}_n$	Set of solar resources in DERA $n$

### 1.1.2 Parameters

Table 1.1 lists the parameters associated with the model in this paper.

Table 1.1: DER Parameters

Symbol	Description	Domain	Unit
$A_t$	Share parameters for inner utility function	$(0,1]$	Unitless
$A_0$	Share parameter for outer utility function	$(0,1]$	Unitless
$B_{tt'}$	Parameters for creating combinations of goods	Real numbers	Unitless

Symbol	Description	Domain	Unit
$C_{k,t}$	The marginal cost of each distributed resource $k$ at time $t$	Nonnegative numbers	\$/MWh
$E_k^{\text{init}}$	Initial SoC of battery $k$	Nonnegative numbers	MWh
$E_k^{\text{min}}$	Lower limits of battery state of charge (SoC)	Nonnegative numbers	MWh
$E_k^{\text{max}}$	Upper limits of battery SoC	Nonnegative numbers	MWh
$P_{k,t}^{\text{max,dr}}$	Upper limits for demand response of DRR	Nonnegative numbers	MW
$P_{k,t}^{\text{max,load}}$	Upper load limits of load	Nonnegative numbers	MW
$P_{k,t}^{\text{min,dr}}$	Lower limits for demand response of DRR	Nonnegative numbers	MW
$P_{k,t}^{\text{min,load}}$	Lower load limits of DRR	Nonnegative numbers	MW
$P_k^{\text{max,c}}$	Upper limits for battery charging	Nonnegative numbers	MW
$P_k^{\text{max,d}}$	Upper limits for battery discharging	Nonnegative numbers	MW
$P_k^{\text{max,sol}}$	Upper power limits of solar resources	Nonnegative numbers	MW
$R'$	Transformed elasticity parameter for the outer utility function	$R' \leq 1$ or $R' = \infty$	Unitless
$S'$	Elasticity parameter for the outer utility function	$S' \geq 0$ or $S' = -\infty$	Unitless
$R$	Transformed elasticity parameter for the inner utility function	$R \leq 1$ or $R = \infty$	Unitless
$S$	Elasticity parameter for the inner utility function	$S \geq 0$ or $S = -\infty$	Unitless
$T$	Number of time intervals	Nonnegative numbers	Unitless
$Y$	Total budget	Nonnegative numbers	\$
$\Delta_T$	Duration of intervals	Nonnegative numbers	minutes
$\zeta_k$	Self-discharge rate for energy storage unit $k$	(0,1)	Unitless
$\alpha$	The time smoothing rate	[0,1)	Unitless

Symbol	Description	Domain	Unit
$\beta$	The time preference parameter	$[0, \infty)$	Unitless
$\gamma_n$	Loss factor for DERA $n$	Nonnegative numbers	Unitless
$\varepsilon$	The time smoothing proportion parameter	$[0, \infty)$	Unitless
$\eta_k$	Round-trip efficiency for energy storage unit $k$	$(0,1)$	Unitless
$\lambda^0$	Initial electricity price forecast vector	Nonnegative numbers	\$/MWh
$\lambda^{\text{rel}}$	Retail rate	Nonnegative numbers	\$/MWh
$\lambda_{i,t}^0$	Initial price at bus $i$ at time $t$	Nonnegative numbers	\$/MWh
$\rho^0$	The share of the budget spent on energy at reference point	$(0,1)$	Unitless

### 1.1.3 Variables

Table 1.2 shows the variables associated with the DERA resources.

Table 1.2: DERA Resource Variables

Symbol	Description	Domain	Unit
$c^{\text{bat}}$	Total cost of batteries in each DERA	Reals numbers	\$
$c^{\text{drr}}$	Total cost of DRR in each DERA	Nonnegative numbers	\$
$c^{\text{sol}}$	Total cost of solar in each DERA	0	\$
$c^{\text{tot}}$	Total cost for all distributed energy resource in DERA	Nonnegative numbers	\$
$c_{k,t}$	Device-level cost at time $t$	Real numbers	\$
$c_{o,t}$	Cost block $o$ at time $t$	Nonnegative numbers	\$
$e_{k,t}$	SoC of battery at end of time $t$ for $k \in \mathcal{N}_n^{\text{BAT}}$	$[E_k^{\min}, E_k^{\max}]$	MWh
$oc_{k,t}^{\text{ch,appx}}$	Opportunity cost of charging for battery $k$ at time $t$ with the approximation method	Nonnegative numbers	\$/MWh
$oc_{k,t}^{\text{ch,dtm}}$	Opportunity cost for charging for battery $k$ at time $t$ with the deterministic method	Nonnegative numbers	\$/MWh



Symbol	Description	Domain	Unit
$oc_{k,t}^{\text{ch,opt}}$	Opportunity cost for charging for battery $k$ at time $t$ with the optimal method	Nonnegative numbers	\$/MWh
$oc_{k,t}^{\text{ch}}$	Opportunity cost of charging for battery $k$ at time $t$	Nonnegative numbers	\$/MWh
$oc_{k,t}^{\text{dis,appx}}$	Opportunity cost of discharging for battery $k$ at time $t$ with the approximation method	Nonnegative numbers	\$/MWh
$oc_{k,t}^{\text{dis,dtm}}$	Opportunity cost of discharging for battery $k$ at time $t$ with the deterministic method	Nonnegative numbers	\$/MWh
$oc_{k,t}^{\text{dis,opt}}$	Opportunity cost of discharging for battery $k$ at time $t$ with the optimal method	Nonnegative numbers	\$/MWh
$oc_{k,t}^{\text{dis}}$	Opportunity cost of charging for battery $k$ at time $t$	Nonnegative numbers	\$/MWh
$p_{t'}^{\text{sch,dera}}$	Scheduled power vector of all distributed resources at time $t'$ in real time market	Real numbers	MW
$p_{k,t}$	Output power of solar at time $t$ for $k \in \mathcal{N}_n^{\text{SOL}}$	$[0, P_k^{\text{max,sol}}]$	MW
$p_{k,t}$	Load power of DRR at time $t$ for $k \in \mathcal{N}_n^{\text{DRR}}$	$[P_{k,t}^{\text{min,load}}, P_{k,t}^{\text{max,load}}]$	MW
$p_{k,t}^{\text{base,c}}$	Baseline charging schedule for storage resource at time $t$	Nonnegative numbers	MW
$p_{k,t}^{\text{base,d}}$	Baseline discharging schedule for storage resource at time $t$	Nonnegative numbers	MW
$p_{k,t}^{\text{base,load}}$	Baseline load schedule for DRR $k$ at time $t$	Nonnegative numbers	MW
$p_{k,t}^{\text{base,sol}}$	Baseline solar $k$ schedule at time $t$	Nonnegative numbers	MW
$p_{k,t}^{\text{bat}}$	Net output power of battery resource at time $t$ for $k \in \mathcal{N}_n^{\text{BAT}}$	Real numbers	MW
$p_{k,t}^{\text{c}}$	Charging power of battery resource at time $t$ for $k \in \mathcal{N}_n^{\text{BAT}}$	$[0, P_k^{\text{max,c}}]$	MW
$p_{k,t}^{\text{d}}$	Discharging power of battery resource at time $t$ for $k \in \mathcal{N}_n^{\text{BAT}}$	$[0, P_k^{\text{max,d}}]$	MW

Symbol	Description	Domain	Unit
$p_{k,t}^{\max,\text{bat}}$	Maximum power limit for battery $k$ at time $t$	Nonnegative numbers	MW
$p_{k,t}^{\min,\text{bat}}$	Minimum power limit for battery $k$ at time $t$	Nonpositive numbers	MW
$p_{o,t}$	Power block $o$ at time $t$	Nonnegative numbers	MW
$p_t$	Total output power of each DERA	Real numbers	MW
$p_t^{\text{base,load}}$	The total DRR baseline load for all $k \in \mathcal{K}_n^{\text{DRR}}$	Nonnegative numbers	MW
$p_t^{\text{bat}}$	Total net power of battery in each DERA	Real numbers	MW
$p_t^{\max,\text{bat}}$	Aggregate maximum battery power limit	Nonnegative numbers	MW
$p_t^{\max,\text{drr}}$	Aggregate maximum demand response	Nonnegative numbers	MW
$p_t^{\max,\text{tot}}$	Overall minimum power for battery and DRR aggregated	Nonnegative numbers	MW
$p_t^{\min,\text{bat}}$	Aggregate minimum battery power limit	Nonpositive numbers	MW
$p_t^{\min,\text{drr}}$	Aggregate minimum demand response	Nonpositive numbers	MW
$p_t^{\min,\text{tot}}$	Overall minimum power for battery and DRR aggregated	Real numbers	MW
$p_t^{\text{sch,dera}}$	A vector for scheduled power of all distributed resources in DERA	Real numbers	MW
$p_t^{\text{sch}}$	Total scheduled power of DERA at time $t$	Real numbers	MW
$p_t^{\text{sol}}$	Total output of solar resources in DERA	$[p_t^{\text{sol,min}}, p_t^{\text{sol,max}}]$	MW
$q^0$	The prices vector of the composite energy goods at reference point	Nonnegative numbers	\$/MWh
$q_t^0$	The reference price of composite energy goods at time $t$	Nonnegative numbers	\$/MWh
$w^0$	The consumed quantity vector of the composite energy goods at reference point	Nonnegative numbers	MW

Symbol	Description	Domain	Unit
$w_t$	The consumed quantity of the composite energy good at time $t$	Nonnegative numbers	MW
$w$	A quantity vector consumed of the composite energy good over time horizon	Nonnegative numbers	MW
$w_t^0$	The reference quantity of the composite energy goods at time $t$	Nonnegative numbers	MW
$x_0$	The consumption of anything other than energy	Nonnegative numbers	Unitless
$x_0^0$	The quantity of other goods consumed at reference point	Nonnegative numbers	Unitless
$\delta^-$	Dispatch deviation down from baseline schedule	Nonnegative numbers	MW
$\delta^+$	Dispatch deviation up from baseline schedule	Nonnegative numbers	MW
$\lambda_{i,t}$	Device-level LMPs	Nonnegative numbers	\$/MWh
$\lambda_t^{\text{rel}}$	Retail price	Nonnegative numbers	\$/MWh
$\pi$	Profit for each DERA	Real numbers	\$
$\Delta p_{k,t}^-$	Negative demand response of DRR for $k \in \mathcal{N}_n^{\text{DRR}}$	Nonpositive numbers	MW
$\Delta p_{k,t}^+$	Positive demand response of DRR for $k \in \mathcal{N}_n^{\text{DRR}}$	Nonnegative numbers	MW
$\Delta p_{k,t}^{\text{base}-}$	Negative demand response of DRR from the baseline schedule	Nonpositive numbers	MW
$\Delta p_{k,t}^{\text{base}+}$	Positive demand response of DRR from baseline schedule	Nonnegative numbers	MW
$\Delta U^{\text{out}}$	Decrease in money-valued nested utility function from the baseline to the actual energy consumption	Nonnegative numbers	\$
$\Delta c_t^{\text{rel}}$	Retail charges decrease from baseline to the actual energy consumption	Nonnegative numbers	\$
$MC_{o,t}$	Marginal cost for offer block $o$	Nonnegative numbers	\$/MWh
$MQ_{o,t}$	Marginal quantity for offer block $o$	Reals numbers	MW

Symbol	Description	Domain	Unit
$MQ_{o,t}^{\text{bat}}$	Marginal quantity for demand offers block $o$ of battery	Reals numbers	MW
$MV_{o,t}^{\text{bat}}$	Marginal value at demand offer block $o$ of battery	Nonnegative numbers	\$/MWh

## 2 DER Model Derivations

In the proposed DERA model, various methods are formulated to derive the associated costs for different types of distributed resources. For demand response resources (DRR), a utility-based cost model is derived while for battery resources, opportunity costs are computed in detail.

### 2.1 DRR Utility Function

Demand response can be implemented through many different mechanisms that allow consumers to adjust their energy usage in response to price signals, grid conditions, or other incentives. DRR here participates in the wholesale market directly via the DERA, which is a variant of incentive based DRR. The demand response values then can be determined either by considering demand elasticity as in (Su and Kirschen, 2009) or by randomizing consumer assigned value from a distribution as in (Xu *et al.*, 2024).

To model such price-based response of DRR in the day-ahead SCUC market and the real-time SCED market, a utility function can be used to derive the cost to dispatch DRR. Four utility function forms have previously applied to represent such preference values of electricity consumption over different time periods. That is, the Constant Elasticity of Substitution (CES) functional form, Cobb-Douglas form, Leontief form and linear form.

Instead of using a constant scaling parameter to convert the four utility functions into dollars, the derivation here employs a nested utility function to ensure such conversion. Let  $p_{k,t}$  and  $x_0$  denote the DRR electricity consumptions and other goods separately. Then the value of DRR can be written in various utility function forms  $U(p_{k,t})$  shown in (1) - (4) respectively. As such, each DRR  $k$  is represented by  $U(p_k)$  where  $p_k$  is a vector for the energy consumption at each time interval and  $U(p_k)$  is defined as a money-valued concave cardinal utility function of energy consumption power levels  $p_{k,t}$  during the time intervals  $t$ .

CES form:

$$U(p_k) = \left( \sum_t A_t^{\frac{1}{S}} p_{k,t}^{\frac{S-1}{S}} + A_0^{\frac{1}{S}} x_0^{\frac{S-1}{S}} \right)^{\frac{S}{S-1}}, k \in \mathcal{K}_n^{\text{DRR}} \quad (1)$$

Cobb-Douglas form:

$$U(p_k) = \prod_t p_{k,t}^{A_t} \times x_0^{A_0}, k \in \mathcal{K}_n^{\text{DRR}} \quad (2)$$

Leontief form:

$$U(p_k) = \min \left\{ \frac{p_{k,t}}{A_t}; \forall t \in \mathcal{T} \right\}, k \in \mathcal{K}_n^{\text{DRR}} \quad (3)$$

Linear form:

$$U(p_k) = \sum_t p_{k,t}, k \in \mathcal{K}_n^{\text{DRR}} \quad (4)$$

Where  $S$  is an elasticity parameter,  $A_t$  is a share parameter in the utility functions such that  $\sum_t A_t = 1$ . Note that the parameters  $A_t$ , and  $S$  do not necessarily take the same value in each functional form. Additionally, the Cobb-Douglas, Leontief, and linear utility functions are limiting cases of the CES function, where the elasticity of substitution parameter approaches limits of 1 (moderate elasticity), 0 (completely inelastic, i.e. perfect complements), and  $\infty$  (completely elastic, i.e. perfect substitutes) respectively.

Furthermore, an intermediate commodity can be created using those utility function forms in addition to the primary commodities (i.e., energy consumptions) at each individual time intervals.

To ensure a value function to be added to the market surplus objective, the intermediate commodities is carefully selected as composite energy goods, which is a weighted sum of electricity consumption. And a nested utility function is chosen in such a way as to be expressed by an explicit algebraic formula. That is, a nested generalized CES utility function  $U(x_0, p_{k,1}, \dots, p_{k,T})$  is introduced as follows.

$$U(p_{k,1}, \dots, p_{k,T}) = U^{\text{out}}(x_0, U'(p_{k,1}, \dots, p_{k,T})) \quad (5)$$

Or, equivalently,

$$U(x) = U^{\text{out}}(x_0, U^0) \quad (6)$$

$$U^0 = U'(w) \quad (7)$$

$$w = Bx' \quad (8)$$

Where  $x = (x_0, x')$ ,  $x' = (p_{k,1}, \dots, p_{k,T})$  with  $p_{k,t}$  denoting the quantity of energy consumed at  $t$  and  $x_0$  representing consumption of anything other than energy in the modeled time periods. The smoothing matrix  $B$  is a nonsingular square matrix,  $w$  is a vector representing the quantity consumed of a composite energy good determined by a weighted sum of electricity over  $t'$ , which can be written as following:

$$w_t = \sum_{t'} B_{tt'} x_{t'} \quad (9)$$

Where the entries  $B_{tt'}$  represent the weight of energy consumptions at  $t'$  in the composite electricity consumption good at  $t$ . The inner utility function  $U'$  is the consumption preference over the composite electricity consumption goods at various time periods, while the outer utility function  $U_{\text{out}}$  represents energy consumption over other goods. A log-scaled form for the outer utility function is defined in (10) to serve as a value function for DRR in the market clearing

process, which maximizes the market surplus over producers and consumers with money-denominated cost and value functions.

$$V(x) = Y \log U(x) \quad (10)$$

Where  $Y$  is the budget for DRR to consume all goods. Note that  $V(x)$  still has the same consumption preferences with  $U(x)$  because it's invariant to strictly monotone transformation.

In this project, CES form (1) is used for both inner and outer utility function since it has limiting cases of all three other forms. The choice of parameters, such as the elasticity parameter  $S$  and the share parameter  $A_t$ , specifies its form shown in (2)-(4). For example, when the elasticity of substitution over electricity consumption is set as  $S \in (0, \infty)$ , then the inner utility function can be written as:

$$U'(w) = \left( \sum_t A_t^{1-R} w_t^R \right)^{1/R}, R \in (-\infty, 1) \quad (11)$$

Where  $R = 1 - \frac{1}{S}$  and  $\sum_t A_t = 1, A_t \geq 0$ .

Note that the inner utility function (11) won't be applied when  $R = 0$  (i.e.,  $S = 1$ ). However, the limit is equivalent to the following when  $R \rightarrow 0$ :

$$U'(w) = \prod_t w_t^{A_t} \quad (12)$$

Which is the Cobb-Douglas case. If  $S = 0$  or  $R = -\infty$ , CES function is a Leontief form while a linear form if  $R = 1$  or  $S = \infty$ .

Likewise, the elasticity parameter  $S' \in (0, \infty)$  and a single share parameter  $A_0 \in (0,1)$  are chosen to define the outer utility function  $U^{\text{out}}(x_0, u)$  as a function of  $x_0$  and inner utility function value  $u$  in (13)-(14).

$$U^{\text{out}}(x_0, u) = \left[ A_0^{1-R'} x_0^{R'} + (1 - A_0)^{1-R'} u^{R'} \right]^{\frac{1}{R'}}, R' \neq 0, R' < 1 \quad (13)$$

Where  $R' = 1 - 1/S'$ .

For the Cobb-Douglas case, the outer utility function is defined as when  $R' \rightarrow 0$ :

$$U^{\text{out}}(x_0, u) = x_0^{A_0} u^{1-A_0} \quad (14)$$

The smoothing matrix  $B$  defines composite goods for energy consumption, so that consumers respond to changes in energy prices by shifting consumption away from more expensive periods and towards cheaper periods. For this, a smoothing rate  $\alpha \in [0,1]$  is defined to represent efficiency of shifting consumption at time periods and then a time preference parameter is set by  $\beta = \alpha/(\alpha - 1)^2 \in [0, \infty)$ . A symmetric triangular matrix  $M$  is constructed as following with  $2\beta + 1$  on the diagonal except for  $\beta + 1$  in the first and last diagonal entries and  $-\beta$  just off the diagonal:

$$M = \begin{bmatrix} \beta + 1 & -\beta & 0 & \dots & \dots & \dots & \dots & 0 \\ -\beta & 2\beta + 1 & -\beta & 0 & \dots & \dots & \dots & \vdots \\ 0 & -\beta & 2\beta + 1 & -\beta & 0 & \dots & \dots & \vdots \\ \vdots & 0 & - & 2\beta + 1 & -\beta & 0 & \dots & \vdots \\ \vdots & \dots & 0 & -\beta & 2\beta + 1 & -\beta & 0 & \vdots \\ \vdots & \dots & \dots & 0 & -\beta & 2\beta + 1 & -\beta & \vdots \\ \vdots & \dots & \dots & \dots & 0 & -\beta & 2\beta + 1 & -\beta \\ 0 & \dots & \dots & \dots & \dots & 0 & -\beta & \beta + 1 \end{bmatrix} \quad (15)$$

By inverting the matrix  $M$ , the matrix  $B$  is obtained:

$$B = M^{-1} \quad (16)$$

Based on the definitions for matrix  $B$ , three properties are concluded as: (1)  $B_{t,(t'+1)}/B_{tt'} \approx \alpha$  for  $t' \geq t$ , which means the geometric rate of change in successive period weights is approximately equal to the smoothing rate; (2)  $\beta = 0$  means no time preference, thus all other times are able to substitute equally well for energy consumption at the specific time; (3)  $\beta > 0$  indicates that energy consumption at  $t'$  closer to  $t$  substitutes better for that at  $t$  than does energy at  $t''$  further from  $t$ .

Additionally, the time smoothing proportion parameter  $\varepsilon$  represents the relative value in the current period of load consumed in future (or previous) periods, which can be converted to  $\alpha$ .

$$\alpha = \varepsilon / (1 + \varepsilon) \quad (17)$$

In the model, a known load baseline is considered as the reference point with the price-quantity pair  $(\lambda_{i,t}^0, p_{k,t}^{\text{base,load}})$ . For simplicity, a price vector  $\lambda^0$  is used to represent the energy price at the reference point, and an energy consumption vector  $p_k^{\text{base,load}}$  to represent the load at all time intervals for demand response resource  $k \in \mathcal{K}_n^{\text{DRR}}$ . With chosen elasticity parameters and the matrix  $B$ , the utility functions can be fully specified using the steps below.

- 1) Given the reference consumption and prices for energy at the reference point, calculate the reference quantities ( $w^0$ ) and prices ( $q^0$ ) of composite energy goods:

$$w^0 = B p_k^{\text{base,load}} \quad (18)$$

$$q^0 = B^{-1} \lambda^0 \quad (19)$$

- 2) Then, the expenditure on energy at the reference point is:

$$\sum_t \lambda_{i,t}^0 p_{k,t}^{\text{base,load}} = \sum_t q_t^0 w_t^0 \quad (20)$$

- 3) With a chosen share parameter  $\rho^0$  for the budget spent on energy at the reference point, the total budget  $Y$  is computed as:

$$Y = \frac{1}{\rho^0 \sum_{k,t} \lambda_{i,t}^0 p_{k,t}^{\text{base,load}}} \quad (21)$$

While the expenditure on other goods at the reference point is  $(1 - \rho^0)Y$ .

- 4) By assuming the price of other goods is \$1 per unit, then the quantities of other goods' consumption are computed as:

$$x_0^0 = (1 - \rho^0)Y \quad (22)$$

- 5) The share parameters  $A_t$  for inner utility function at each time interval is:

$$A_t = \frac{(q_t^0)^{1/(1-R)} w_t^0}{\sum_{t'} (q_{t'}^0)^{1/(1-R)} w_{t'}^0} \quad (23)$$

- 6) To set the share parameter  $A_0$  for outer utility function, it can be computed as (24) if it's Cobb-Douglas form ( $S' = 1$ ) or as (25) if it's a CES form:

$$A_0 = 1 - \rho^0, R' = 0 \quad (24)$$

$$A_0 = \frac{\left(\frac{\rho^0}{1 - \rho^0}\right)^{1/(1-R')}}{1 + \left(\frac{x_0^0}{U^0}\right)^{R'/(1-R')}} \quad (25)$$

Where  $U^0$  is the value of the inner utility function at the reference point:

$$U^0 = U'(w^0) \quad (26)$$

In such a way, the constructed utility functions has a consistency property that a consumer choosing consumption quantities  $p_{k,t}$  so as to maximize the utility  $U(p_{k,1}, \dots, p_{k,T})$  subject to the total budget  $Y$  and prices  $\lambda_{i,t}$  will consume the reference quantities if the prices are equal to the reference prices.

With non-energy consumption fixed at the reference value, the actual energy consumption of DRR at each time interval is equal to the reference value minus the flexible energy  $p_{k,t}^{\text{drr}}$  provided by demand response, i.e.,  $p_{k,t} = p_{k,t}^{\text{base,load}} - p_{k,t}^{\text{drr}}$ .

To compute the demand response cost, first the changes in utility functions ( $\Delta U^{\text{out}}$ ) can be written with reference and actual energy consumption vector:

$$\Delta U^{\text{out}} = Y \log U(p_k^{\text{base,load}}) - Y \log U(p_k), \forall k \in \mathcal{K}_n^{\text{DRR}} \quad (27)$$

To suppress DRR's incentives to increase its load significantly without any penalties, an additional retail rate term ( $\lambda^{\text{rel}}$ ) is introduced, which is computed as a constant equal to the average retail rate across the U.S electricity market is assumed with elasticity of 0.1. The associated retail price  $\lambda_t^{\text{rel}}$  is obtained by normalizing based on the average energy consumption.

$$\lambda_t^{\text{rel}} = T \frac{\lambda^{\text{ret}} p_{k,t}^{\text{base,load}}}{p_t^{\text{base,load}}} \quad (28)$$

Then, at time  $t$ , with demand response  $p_{k,t}^{\text{drr}}$  dispatched, the change in retail costs  $\Delta c_t^{\text{rel}}$  is computed as:

$$\Delta c_t^{\text{rel}} = \lambda_t^{\text{rel}} [-p_{k,t}^{\text{drr}} - p_{k,t}^{\text{base,load}}] \left(\frac{\Delta_T}{60}\right) \quad (29)$$



Then the demand response cost is the signed decrease in the money-valued function  $U$  from the baseline energy consumption to actual consumption minus the signed decrease in retail charges paid by the consumer. To be more consistent with the time-separable offer format required by typical wholesale electricity markets, we define a time-separable approximation of the demand response cost by considering the sum of differences in the money-valued utility over single interval consumption changes.

$$c_k = \sum_t \left( V(p_{k,t}^{\text{base,load}}) - V(p_{k,t}^{\text{base,load}} - \mathbf{1}_t p_{k,t}^{\text{drr}}) \right) - \Delta c_t^{\text{rel}} \quad (30)$$

Where  $\mathbf{1}_t$  is a vector equal to 0 except for 1 in entry  $t$ .

Note that each DRR can responds to prices bidirectionally, which means it could be a positive demand response  $\Delta p_{k,t}^+$  or a negative demand response  $\Delta p_{k,t}^-$ . Then the total demand response can be written as:

$$p_{k,t}^{\text{drr}} = \Delta p_{k,t}^+ - \Delta p_{k,t}^- \quad (31)$$

The LMP-based revenue for DRR is:

$$\pi_k = \sum_t \lambda_{i,t} \Delta p_{k,t}^+, \forall k \in \mathcal{K}_n^{\text{DRR}}, \forall t \in T \quad (32)$$

## 2.2 Storage Opportunity Cost

BESS cost calculations commonly assume a zero marginal cost dispatch, which is also considered as the explicit cost way for BESS. However, the opportunity cost of BESS dynamically determines the potential value of the stored energy at each time. For example, BESS which chooses to discharge at the current time instead of discharging at a higher price in the future, has some opportunity cost occurred for this charging behaviors. Different methods have been applied to capture the optimal bidding strategies of BESS to maximize its profit (Biggins *et al.*, 2022; He *et al.*, 2018; Padmanabhan *et al.*, 2019; Shabani *et al.*, 2024). In this paper, three methodologies have been implemented to derive the opportunity cost for BESS, i.e., “approximation method”, “optimization method” and “deterministic method”, respectively.

### 2.2.1 Approximation Method

For the “approximation method”, at each time period  $t$ , an observation window with time interval of  $6\Delta_T$  are constructed over the whole-time horizon, i.e.,  $[t - 3\Delta_T, t + 3\Delta_T]$ . The opportunity cost for charging and discharging can be obtained following:

$$oc_{k,t}^{\text{ch,appx}} = \begin{cases} \min(\lambda_{k,t} | t = t - 3\Delta_T, \dots, t + 3\Delta_T) & \text{if charging at time } t \\ \max(\lambda_{k,t} | t = t - 3\Delta_T, \dots, t + 3\Delta_T) * \eta_k & \text{if discharging at time } t \end{cases} \quad (33)$$

$$oc_{k,t}^{\text{dis,appx}} = \begin{cases} \min(\lambda_{k,t} | t = t - 3\Delta_T, \dots, t + 3\Delta_T) / \eta_k & \text{if charging at time } t \\ \max(\lambda_{k,t} | t = t - 3\Delta_T, \dots, t + 3\Delta_T) & \text{if discharging at time } t \end{cases} \quad (34)$$

### 2.2.2 Optimization Method

For the “optimization method”, the following steps are taken to calculate the opportunity costs for charging ( $oc_{k,t}^{\text{ch,opt}}$ ) and discharging ( $oc_{k,t}^{\text{dis,opt}}$ ) at time  $t$  :

- 1) If charging at time  $t$ , the opportunity cost for charging is obtained by maximizing the battery’s profit with fixed charging amount of  $p_{k,t}^c, \forall k \in \mathcal{K}_n^{\text{BAT}}$ ;
- 2) If discharging at time  $t$ , the opportunity cost for discharging is obtained by maximizing the battery’s profit with fixed discharging amount of  $p_{k,t}^d, \forall k \in \mathcal{K}_n^{\text{BAT}}$ .

### 2.2.3 Deterministic Method

For the “deterministic method”, battery’s opportunity cost is computed based on the DERA baseline schedule for BESS, which considers the closest upcoming profitable opportunity.

At time  $t$ , BESS has three possible actions, i.e., charging, discharging, or nothing. Over the whole -time horizon, there are multiple charging or discharging in a row, and each decision needs to be dynamically adjusted based on the sequence of upcoming events. Let  $t_{\text{ch}}, t_{\text{dis}}, t_{\text{ch}}^{\text{nx}}, t_{\text{dis}}^{\text{nx}}$  represent the time for current charging/discharging cycle and the next immediate charging/discharging events.

- 1) At time  $t = t_{\text{ch}}$  with a scheduled withdrawal (charging), the opportunity cost for charging and discharging are:

$$oc_{k,t_{\text{ch}}}^{\text{ch,dtm}} = \min(\min(\lambda_{k,t}|t = 0, \dots, t_{\text{ch}} - 1, t_{\text{ch}} + 1, \dots, t_{\text{dis}} - 1), \eta_k \lambda_{k,t_{\text{dis}}}) \quad (35)$$

$$oc_{k,t_{\text{ch}}}^{\text{dis,dtm}} = -\lambda_{k,t_{\text{ch}}} + \min\left(\frac{\lambda_{k,t}}{\eta_k} \middle| t = t_{\text{ch}} + 1, \dots, t_{\text{dis}} - 1\right) \quad (36)$$

The opportunity cost for charging in (35) reflects cost of missing a better opportunity to either charge at a lower price at other times than the current time before discharging happens or to wait and discharge at a higher price in the future with a discount of round-trip efficiency. The first term in opportunity cost for discharging at a scheduled charging event in (36) represents the potential loss if discharging now at the current period, while the second term represents the most profitable future discharge opportunity which could be missed by discharging now.

- 2) At time  $t = t_{\text{dis}}$  with a scheduled power injection (discharging), opportunity cost for charging and discharging are:

$$oc_{k,t_{\text{dis}}}^{\text{ch,dtm}} = -\lambda_{k,t_{\text{dis}}} + \max(\lambda_{k,t}|t = t_{\text{ch}}^{\text{nx}} + 1, \dots, t_{\text{dis}}^{\text{nx}} - 1) \quad (37)$$

$$oc_{k,t_{\text{dis}}}^{\text{dis,dtm}} = \max\left(\max(\lambda_{k,t}|t = t_{\text{ch}}^{\text{nx}} + 1, \dots, t_{\text{dis}}^{\text{nx}} - 1), \frac{\lambda_{k,t_{\text{ch}}^{\text{nx}}}}{\eta_k}\right) \quad (38)$$

The opportunity cost for charging in (37) represents the potential lost revenue from not discharging at the current price and the best future charging opportunity while the opportunity cost for discharging in (38) represent the chance to discharge at a higher price in the future or to charge at a lower price and discharge at a later, more profitable time.

- 3) For time periods before the first charging time  $t < t_{\text{ch}}^{\text{fir}}$ , the opportunity costs are:

$$oc_{k,t}^{\text{ch,dtm}} = \max\left(\eta_k \max(\lambda_{k,t'} | t' = t + 1, \dots, t_{\text{ch}}^{\text{fir}} - 1), \lambda_{k,t}^{\text{fir}}\right), \forall t = 0, 1, \dots, t_{\text{ch}}^{\text{fir}} - 1 \quad (39)$$

$$oc_{k,t}^{\text{dis,dtm}} = \frac{\min(\lambda_{k,t'} | t' = 0, \dots, t - 1)}{\eta_k}, \forall t = 0, 1, \dots, t_{\text{ch}}^{\text{fir}} - 1 \quad (40)$$

At time  $t$  before the first charging happens, the opportunity cost of charging in (39) represents the potential lost benefits if charging now but not waiting until a future time before the first charging time when you could discharge at a higher price with round-trip efficiency accounted. And the opportunity cost for discharging before first charging happens in (40) reflect the lost benefits if discharging now but without having stored energy at a cheaper price.

4) For time periods after the last discharging time  $t_{\text{dis}}^{\text{lst}}$ , the opportunity costs are:

$$oc_{k,t}^{\text{ch,dtm}} = \eta_k \max(\lambda_{k,t'} | t' = t + 1, \dots, T - 1), t_{\text{dis}}^{\text{lst}} < t < T \quad (41)$$

$$oc_{k,t}^{\text{dis,dtm}} = \frac{\min(\lambda_{k,t'} | t' = t + 1, \dots, T - 1)}{\eta_k}, t_{\text{dis}}^{\text{lst}} < t < T \quad (42)$$

At time  $t$  after the final discharging until the end of the time horizon, the opportunity cost of charging in (41) is the potential benefit loss adjusted by round-trip efficiency by not discharging at a potentially higher price between  $t$  and the end of the time horizon if charging at  $t$ . And the opportunity cost for discharging is the potential revenue loss from discharging now at a low price rather than waiting for a better price opportunity in the future, adjusted for efficiency.

5) For other time intervals without ongoing charging or discharging between first charging and last discharging period, the opportunity costs for charging and discharging are:

$$oc_{k,t}^{\text{ch,dtm}} = \max(\eta_k \max(\lambda_{k,t'} | t' = t + 1, \dots, t_{\text{ch}}^{\text{nx}} - 1), \lambda_{k,t}), \forall t \in (t_{\text{ch}}^{\text{fir}}, t_{\text{ch}}^{\text{lst}}) \quad (43)$$

$$oc_{k,t}^{\text{dis,dtm}} = \min\left(\min(\lambda_{k,t'} | t' = t + 1, \dots, t_{\text{dis}}^{\text{nx}} - 1) \eta_k, \lambda_{k,t}^{\text{nx}}\right), \forall t \in (t_{\text{ch}}^{\text{fir}}, t_{\text{ch}}^{\text{lst}}) \quad (44)$$

The opportunity costs of charging in (43) is the value missed if not charging during this hour, considering the best potential value which could have realized in the future. The value of not discharging now should be compared to the potential future value which could be realize if discharged at a later time, as shown in (44).

The charging cost for battery  $k$  in DERA  $n$  (i.e.,  $k \in \mathcal{K}_n^{\text{BAT}}$ ) is:

$$c_k^{\text{ch}} = \sum_t oc_{k,t}^{\text{ch}} p_{k,t}^{\text{c}}, \forall k \in \mathcal{K}_n^{\text{BAT}} \quad (45)$$

The discharging cost for battery  $k$  in DERA  $n$  is:

$$c_k^{\text{dis}} = \sum_t oc_{k,t}^{\text{dis}} p_{k,t}^{\text{d}}, \forall k \in \mathcal{K}_n^{\text{BAT}} \quad (46)$$

The LMP-based revenue for battery  $k$  in DERA  $n$  with the loss factor  $\gamma_n$  is:

$$\pi_k = (1 + \gamma_n) \sum_t \lambda_{i,t} (p_{k,t}^{\text{c}} - p_{k,t}^{\text{d}}), \forall k \in \mathcal{K}_n^{\text{BAT}}, \forall t \in T \quad (47)$$

### 3 Constraints

For each DERA, there are some constraints on the three resource types modeled in this document. Solar is considered as a non-dispatchable generator, BESS is modeled by a battery storage formulation with round-trip efficiency and per-period energy leakage. For the price-responsive demand, (also called demand response in the document), a value function is introduced to transform the utility function to an economic value, where the demand elasticity is modeled using the utility function. Among the three DER types, DRR has an explicit cost function in the aggregator's DER dispatch model while rooftop solar is assumed to have no explicit cost. For BESS, explicit cost is used to get the baseline schedule of DERA when DERA maximizes its profit. Opportunity cost-based cost is used in the dispatch model to help minimize cost of the overall aggregation.

The following subsections describe individual features of the sub-components of DERA. For an aggregation  $n \in \mathcal{N}$ , each DER resource  $k \in \mathcal{K}$  is considered a sub-component.

#### 3.1 Aggregation

For each DERA  $n$  in the market, each sub-component  $k \in \mathcal{K}$  is distributed at buses ( $i$ ). The device-level prices can be represented with the local marginal price ( $\lambda_{i,t}$ ) by mapping the device to its locations. Then a profit-maximizing optimization problem is formulated using the Pyomo library (Hart *et al.* 2011) as following. With  $\lambda_{i,t}$  as inputs, Pyomo model outputs the power output schedules  $p_{k,t}$  for solar and DRR as well as charging and discharging schedule ( $p_{k,t}^c, p_{k,t}^d$ ) for battery resources. Thus, the net output power of each battery resource is  $p_{k,t} = p_{k,t}^d - p_{k,t}^c$ ,  $k \in \mathcal{K}_n^{\text{BAT}}$  and the total power output of DERA ( $p_t$ ) can be obtained at each time with equation (49).

$$\max \pi = \sum_{t \in \mathcal{T}} \sum_{k \in \mathcal{K}_n \cap \mathcal{K}_i^{\text{bus}}} (\lambda_{i,t} p_{k,t} - c_{k,t}) \quad (48)$$

Subject to:

$$p_t = (1 + \gamma_n) \left( \sum_{k \in \mathcal{K}_n^{\text{DRR}}} \Delta p_{k,t}^+ + \sum_{k \in \mathcal{K}_n^{\text{SOL}} \cup \mathcal{K}_n^{\text{BAT}}} p_{k,t} \right) \quad (49)$$

$$\{p_{k,t}, c_{k,t}\} \in W^k \quad (50)$$

Where  $c_{k,t}$  represents the cost associated with power outputs for each resource  $k \in \mathcal{K}$  at time  $t$  and  $\Delta p_{k,t}^+$  for the positive demand response from load  $k$ . For solar type,  $c_{k,t} = 0$ , while it can be 0 for explicit cost method or opportunity cost-based method for battery type. The last constraint (50) indicates all sub-component resources need to satisfy their own operational constraints with  $W^k$  representing the feasible set for resource  $k$ .

#### 3.2 Individual Resources

Each individual resource in DERA needs to operate within its own constraints. For example, each resource has lower and upper output constraints. Battery resource must manage the SoC associated with charging/discharging cycles. The following sections discuss the constraints specifically to each resource type.

### 3.2.1 Solar

For the set of solar resources  $k \in \mathcal{K}_n^{\text{SOL}}$  in each aggregation  $n \in \mathcal{N}$ , every individual unit  $k$  has a power output of  $p_{k,t}$  at time  $t$ , which constrained by 0 and maximum power output constraints.

$$0 \leq p_{k,t} < P_k^{\text{max,sol}}, \forall k \in \mathcal{K}_n^{\text{SOL}} \quad (51)$$

The cost of solar generation  $c_{k,t}$  in constraint (50) can be written as:

$$c_{k,t} = C_{k,t} p_{k,t}, \forall k \in \mathcal{K}_n^{\text{SOL}} \quad (52)$$

The implementation assumes  $C_{k,t} = 0$  for solar resources, i.e., zero marginal dispatch cost.

### 3.2.2 Battery

Storage resources  $k \in \mathcal{K}_n^{\text{BAT}}$  are modeled using round-trip efficiency  $\eta_k$ , maximum discharge rate  $P_k^{\text{max,d}}$ , self-discharge rate  $\zeta_k$ , maximum charge rate  $P_k^{\text{max,c}}$ , initial state of charge  $E_k^{\text{init}}$ , required end state of charge  $E_k^{\text{end}}$ , and minimum/maximum state-of-charge  $E_k^{\text{min}}$  and  $E_k^{\text{max}}$ .

For each storage unit at time  $t$ , the power output  $p_{k,t}$  is the net power between discharge  $p_{k,t}^{\text{d}}$  and charge  $p_{k,t}^{\text{c}}$ , as shown in (53). Both charging and discharging power have its own lower and upper bounds in (54) and (55). To prevent simultaneous charging and discharging by a single resource, the product of charge and discharge is introduced to be less than or equal to 0 in (56).

For the state of charge (SoC) level, (57)-(59) represent the lower and upper bounds while (60) indicates the SoC level changes from  $e_{k,(t-1)}$  to  $e_{k,t}$  at each time interval  $\Delta_T$  (minutes).

$$p_{k,t} = p_{k,t}^{\text{d}} - p_{k,t}^{\text{c}} \quad (53)$$

$$0 \leq p_{k,t}^{\text{d}} \leq P_k^{\text{max,c}} \quad (54)$$

$$0 \leq p_{k,t}^{\text{c}} \leq P_k^{\text{max,c}} \quad (55)$$

$$p_{k,t}^{\text{d}} p_{k,t}^{\text{c}} \leq 0 \quad (56)$$

$$E_k^{\text{min}} \leq e_{k,t} \leq E_k^{\text{max}} \quad (57)$$

$$e_{k,0} = E_k^{\text{init}} \quad (58)$$

$$e_{k,T} \geq E_k^{\text{end}} \quad (59)$$

$$e_{k,t} = e_{k,t-1} * (1 - \zeta_k) + \left(\frac{\Delta_T}{60}\right) * (\eta_k * p_{k,t}^{\text{c}} - p_{k,t}^{\text{d}}), \forall k \in \mathcal{K}_n^{\text{BAT}}, \forall t \in T \quad (60)$$

The cost for BESS in constraint (50) can be computed with either explicit charging/discharging cost in (61) assuming its zero marginal cost. Or it is the sum of the opportunity costs for charging and discharging in (62).

$$c_{k,t} = 0, \forall k \in \mathcal{K}_n^{\text{BAT}}, \forall t \in T \quad (61)$$

$$c_{k,t} = oc_{k,t}^{\text{ch}} p_{k,t}^{\text{c}} + oc_{k,t}^{\text{dis}} p_{k,t}^{\text{d}}, \forall k \in \mathcal{K}_n^{\text{BAT}}, \forall t \in T \quad (62)$$

Where  $oc_{k,t}^{\text{ch}}, oc_{k,t}^{\text{dis}}$  denotes the charging/discharging cost from approximation method in (33)-(34), the optimization method, or from the deterministic method in (35)-(44).

### 3.2.3 Demand Response Resource

In the optimization model, to maintain consistency with power injection provided by other resources, we model the arithmetic relationship between price-responsive demand  $p_{k,t}$ , baseline energy consumption  $p_{k,t}^{\text{base,load}}$ , and the demand response resource  $p_{k,t}^{\text{drr}}$  that represents a virtual “injection” of power. To transform the value function to be included as DRR costs like other resources in the objective, the nested value function is subtracted from the value associated with DRR’s baseline consumption  $p_t^{\text{base,load}}$ :

$$c_{k,t} = U^{\text{out}}(p_{k,t}^{\text{base,load}}) - U^{\text{out}}(p_{k,t}), \forall k \in \mathcal{K}_n^{\text{DRR}} \quad (63)$$

This transformation does not affect the resource’s optimal dispatch since it is a change in the objective function by a constant value, but it allows DRR to be associated with a positive cost function when demand is curtailed and a positive value function when demand is increased.

For the constrains of DRR’s energy, the lower and upper limits of  $P_k^{\text{min,load}}$  and  $P_k^{\text{max,load}}$  are applied in (64) while the relationship between available demand response and the energy consumption is show in (65).

$$P_{k,t}^{\text{min,load}} \leq p_{k,t} \leq P_{k,t}^{\text{max,load}}, k \in \mathcal{K}_n^{\text{DRR}}, \forall t \in \mathcal{T} \quad (64)$$

$$p_{k,t} = p_{k,t}^{\text{base,load}} - (\Delta p_{k,t}^+ - \Delta p_{k,t}^-), k \in \mathcal{K}_n^{\text{DRR}}, \forall t \in \mathcal{T} \quad (65)$$

The upper and lower limits can be adjusted, for example, to allow demand response up to a percentage  $\vartheta \in (0,1)$  of load, or to allow additional energy consumption as following.

$$P_{k,t}^{\text{min,drr}} = (1 - \vartheta)p_{k,t}^{\text{base,load}}, k \in \mathcal{K}_n^{\text{DRR}}, t \in \mathcal{T} \quad (66)$$

$$P_{k,t}^{\text{max,drr}} = (1 + \vartheta)p_{k,t}^{\text{base,load}}, k \in \mathcal{K}_n^{\text{DRR}}, t \in \mathcal{T} \quad (67)$$

## 3.3 Objective Functions

In the SCUC-DER project, various models are configured at different steps to help dispatch energy resources. The difference between them lies in either the objective functions they aim to optimize or the constraints they must include. Specifically, the profit maximizing operation, minimum cost dispatch following operation, and dispatch following operation are implemented. In this section, we will discuss the optimization models used for model operations.

### 3.3.1 Profit Maximization

In Section 3.1, the profit maximization problem for each DERA is formulated with (48)-(50). Alternatively, the profit maximization operation takes prices  $\lambda$  from relaxed SCUC as inputs and gives a commitment schedule for generators as outputs, with objective rewritten as:

$$\pi = \sum_t \lambda_t p_t - \sum_{k,t} c_{k,t} \quad (68)$$

Where  $\sum_t \lambda_t p_t$  represents the LMP based revenue while  $\sum_{k,t} c_{k,t}$  for the total costs of distributed energy resources.

### 3.3.2 Cost Minimization

Instead of maximizing the profit, this method ignores the revenues but minimize the total costs  $c^{\text{tot}}$ . The objective function can be written as:

$$c^{\text{tot}} = \sum_{k,t} c_{k,t} \quad (69)$$

### 3.3.3 Deviation Minimization

Unlike the profit-maximization or the cost minimization objective functions, the deviation minimization aims to minimize the total distance between the actual DERA dispatch and baseline schedules without considering revenues or cost at all. The corresponding optimization problem can be written as:

$$\delta^{\text{tot}} = \sum_t (\delta_t^+ + \delta_t^-) \quad (70)$$

$$\delta_t^+ - \delta_t^- = p_t - p_t^{\text{sch}} \quad (71)$$

$$\delta_t^+ \geq 0 \quad (72)$$

$$\delta_t^- \geq 0 \quad (73)$$

## 4 Model Specification

The following section describes the full model specifications for different functional applications of the DERA model. That is, various configurations of the constraints formulated in Section 2 are implemented during various phases of DERA operations in the SCUC-DER simulation platform. Briefly, these configurations are summarized in Table 4.1 below.

Table 4.1: Model specifications

Specification Name	Description	Equations
Profit	All resources receive LMP revenue and incur dispatch costs.	Objective: $\max \pi$ DERA: (49) Solar: (51)-(52) Battery: (53)-(61) DRR: (63)-(65)
Schedule	Ignores resource cost and minimizes the deviation from ISO dispatch schedule.	Objective: $\min \delta^{\text{tot}}$ DERA: (49) Solar: (51)-(52) Battery: (53)-(61) DRR: (63)-(65)
Cost	Minimize cost to dispatch resources, including an explicitly defined cost for storage. No assumed LMP revenue.	Objective: $\min c^{\text{tot}}$ DERA: (49) Solar: (51)-(52) Battery: (53)-(61) DRR: (63)-(65)

Specification Name	Description	Equations
Offer	Minimize cost to dispatch resources, assuming LMP revenue for storage. No assumed LMP revenue for other resources.	Objective: $\min c^{\text{tot}}$ DERA: (49) Solar: (51)-(52) Battery: (53)-(60), (62) DRR: (63)-(65)

## 4.1 Aggregation Methods

To offer its component DERs into the wholesale market, the DERA needs to format its offer specifically according to the market operator. For an aggregation of rooftop solar, the offer can be obtained by simply summing up the total available solar output. It is more complicated for battery storage and price-responsive demand, which may have intertemporal price dependencies. That is, the value of storage will depend on its round-trip efficiency and the expected prices that stored energy can be sold. In our model of Cobb-Douglas utility for price responsive demand, changes in consumption in any period can affect the utility of energy consumption other periods. Given these complications, the following models may not necessarily identify the optimal aggregation, but instead they are proposed as reasonable approaches to the DERA's problem.

Given the price forecasts from a relaxed SCUC problem, each DERA computes a baseline schedule to maximize its profit. Two methods are proposed to compute the aggregation cost curve based on the baseline schedule, an optimization-based method in Section 4.1.2 and a cost-based method in Section 4.1.3.

Based on the DERA baseline schedule, the size of offer blocks ( $o \in O, |O| \leq 10$ ) is first determined in MW at each time interval, and the minimum offer satisfies  $p_{o,t} \geq 0.1\text{MW}$  in accordance with FERC Order 2222. When the baseline DERA schedule is less than the 0.1 MW, no block is created.

### 4.1.1 Baseline Schedule

The baseline schedule is obtained with DERA to maximize the total profit over the time horizon based on the model configuration as shown in Table 4.2. The following steps are taken:

- 1) Solve the relaxed SCUC problem in Prescient/Egret (Knueven, *et al.*, 2022) to get the initial price forecasts  $\lambda^0$ ;
- 2) Taking  $\lambda^0$  as the inputs for the day-ahead market, DERA solves the profit maximization problem which gives a baseline schedule vector  $p_t^{\text{sch,dera}}$ , as well as baseline schedules for each distributed resource,  $p_{k,t}^{\text{base,load}}$ ,  $\Delta p_{k,t}^{\text{base+}}$ ,  $\Delta p_{k,t}^{\text{base-}}$ ,  $p_{k,t}^{\text{base,c}}$ , and  $p_{k,t}^{\text{base,d}}$ .

Based on the baseline schedule in day-ahead market, a SCED baseline  $p_t^{\text{sch,dera}}$  for the real-time market can be derived using a splines interpolation method. For example, the SCED baseline for solar can be calculated shown in (74). The same approach can be used to compute the baseline schedule for battery and DRR respectively.



$$p_{k,t'}^{\text{base,sol}} = p_{k,t}^{\text{base,sol}} + b(t' - t) + c(t' - t)^2 + d(t' - t)^3 \quad (74)$$

$$p_{k,(t+1)}^{\text{base,sol}} = p_{k,t}^{\text{base,sol}} + b + c + d$$

where  $t$  and  $t'$  denote the time in day-ahead and real time respectively.

Furthermore, with the baseline schedule obtained, the lower and upper bounds of capacity for each resource can be obtained. For the DRR resource, equations (66)-(67) are used while for BESS, the following formula is used to compute its lower and upper bounds of capacity:

$$p_{k,t}^{\text{min,bat}} = -\max\left(0, \min\left(p_{k,t}^{\text{base,c}}, \frac{e_{k,t} - e_{k,(t-1)}}{\eta_k \Delta_T/60}\right)\right), \forall k \in \mathcal{K}_n^{\text{BAT}} \quad (75)$$

$$p_{k,t}^{\text{max,bat}} = \max\left(0, \min\left(p_{k,t}^{\text{base,d}}, \frac{e_{k,t}}{\Delta_T/60}\right)\right), \forall k \in \mathcal{K}_n^{\text{BAT}} \quad (76)$$

Then for the battery resource type with multiple BESS, the overall bounds at time  $t$  is:

$$p_t^{\text{min,bat}} = \sum_{k \in \mathcal{K}_n^{\text{BAT}}} p_{k,t}^{\text{min,bat}}, \forall t \quad (77)$$

$$p_t^{\text{max,bat}} = \sum_{k \in \mathcal{K}_n^{\text{BAT}}} p_{k,t}^{\text{max,bat}}, \forall t \quad (78)$$

For the DRR resource type, the overall bounds at time  $t$  is:

$$p_t^{\text{min,drr}} = 0, \forall t \quad (79)$$

$$p_t^{\text{max,drr}} = \sum_{k \in \mathcal{K}_n^{\text{DRR}}} \Delta p_{k,t}^{\text{base+}}, \forall t \quad (80)$$

Note that the lower bound for DRR capacity is implemented as 0, as negative demand response is not compensated.

Table 4.2: Baseline model configuration

Model component	Equations
DERA	(49)
Solar	(51)-(52)
Battery	(53)-(61)
Demand Response	(63)-(65)

### 4.1.2 Optimization-based Method

First, the offer block sizes are first determined within the maximum of 10 blocks before aggregating the cost offers. In the optimization-based aggregation method, the offer blocks for dispatch quantities can be decided at time  $t$  as following:

- 1) The total bounds without solar resource are computed with  $p_t^{\min, \text{tot}} = p_t^{\min, \text{drr}} + p_t^{\min, \text{bat}}$  and  $p_t^{\max, \text{tot}} = p_t^{\max, \text{drr}} + p_t^{\max, \text{bat}}$  respectively.
- 2) Then the number of offer blocks  $o_t$  is determined by  $o_t = \min \left( 10, \frac{(p_t^{\max, \text{tot}} - p_t^{\min, \text{tot}})}{0.1} \right)$ . An array of power blocks can be obtained by evenly dividing the upper and lower bounds with  $o_t$ . The values in each block represents the dispatch quantities without solar resources.
- 3) If the number of offer blocks for combined BESS and DRR is not 0, then the upper limits from solar will be added to all the bins with the available solar output. Multiplying the values in each bin by  $(1 + \gamma_n)$  will generate the offer block sizes in MW for the aggregation cost offers.

Note that if  $(p_t^{\max, \text{tot}} - p_t^{\min, \text{tot}} < 0.1)$  in step 2, then the number of bins for combined BESS and DRR is 0. There will be no bidding blocks for BESS or DRR, only the quantities for solar resources will be included in the aggregation offer blocks.

Based on the offer block for available DERA dispatch quantities, the optimization-based method generates the offer curves by optimizing the dispatch of resources at each time step. At time  $t$ , the total power of block offers is set to the baseline power  $p_t$  and the model solves the DERA cost minimization problem ("Offer" model in Table 4.1) to determine the associated cost ( $c_{o,t}$ ) for each block at each time interval.

The following algorithm shows the steps to obtain the block cost  $c_{o,t}$  for each time interval  $t$ :

- 1) Fix DERA dispatch  $\sum_o p_{o,t} = p_t$ ;
- 2) A cost-minimizing of DERA (the "Offer" Model in Table 4.1) is solved across all time interval based on the predicted price  $\lambda_t$  from the relaxed SCUC problem. The associated objective as shown in (69) is recorded as the resource cost block  $c_{o,t}$  in units of dollars.
- 3) Also at each time interval, a zero-dispatch case ( $p_t = 0$ ) is introduced to obtain an initial block cost ( $c_{0,t}$ ).

Repeat step 1-3 for all blocks and all-time intervals. From these costs, an offer of marginal quantity (MQ) and marginal cost (MC) in each interval is constructed as following with length of  $\Delta_T/60$  hours. MQ is in units of MW and MC is in units of \$/MWh.

$$MQ_{o,t} = p_{o,t} - p_{(o-1),t}; \quad p_{o,t} > 0 \quad (81)$$

$$MC_{o,t} = \frac{c_{o,t} - c_{(o-1),t}}{(\Delta_T/60) (p_{o,t} - p_{(o-1),t})} \quad (82)$$

For the batter storage system, battery charging is constructed as a demand offer with a MQ and marginal value (MV), which are showed as following.

$$MQ_{o,t}^{\text{bat}} = p_{o,t} - p_{(o-1),t}; p_{o,t} < 0 \quad (83)$$

$$MV_{o,t}^{\text{bat}} = \frac{c_{(o-1),t} - c_{o,t}}{(\Delta_T/60) (p_{(o-1),t} - p_{o,t})} \quad (84)$$

These offers are adjusted to ensure convexity, such that marginal cost offers are in increasing order and marginal value offers are in decreasing order.

### 4.1.3 Cost-based Method

Similarly to the optimization-based method, the offer block size is determined first from the DERA baseline schedule. In this method, an additional step is needed to divide the blocks by resource type. This gives dispatch quantities  $p_{o,t}$  where resource is composed entirely of either solar, battery or DRR. Note that the minimum power limits of 0.1 MW are not enforced on individual resource. However, it will apply to the aggregated blocks later.

For solar at most one single block is included, depending on the available solar power from (51). If  $p_t^{\text{sol}} = 0$ , the solar block will be omitted.

For battery, although its dispatch could be divided into multiple blocks, the cost would be the same with a single battery type. Thus, at most a single block is employed to represent charging and discharging block respectively. Block sizes are set to the charge or discharge rate, limited by available SoC level based on (77) and (78) :

$$p_{o,t}^d = \sum_{k \in \mathcal{K}_n^{\text{BAT}}} p_{k,t}^{\text{min,bat}} \quad (85)$$

$$p_{o,t}^c = \sum_{k \in \mathcal{K}_n^{\text{BAT}}} p_{k,t}^{\text{max,bat}} \quad (86)$$

The remaining blocks are assigned to DRR. If available DRR is below 0.2 MW, only one block is assigned to DRR. Otherwise, DRR will be divided into evenly sized blocks such that the total number of blocks, include solar, battery discharge, and DRR are less than or equal to 10.

To compute the resource-based costs separately for each block, the following logics are employed:

- 1) If it's a solar resource, the associated cost  $c_{o,t} = 0$ ;
- 2) If it's a battery resource,  $c_{o,t}$  is obtained as the form of opportunity cost in equations (33)-(34) or (35)-(44);
- 3) If it's a DRR resource,  $c_{o,t}$  is determined by equations (30).

Then the blocks will be ordered by increasing marginal cost regardless of resource type. If there's block with power less than 0.1 MW, it will be merged to the adjacent block. If after

merging, the sum of available MW at a time interval is still less than 0.1 MW, then no offer will be submitted for that interval.

## 4.2 Disaggregation Methods

After SCUC market clearing process is completed, the algorithms above are used to generate updated offers in a real-time SCED simulation. For real-time offers, the baseline case  $p_{t'}^{\text{sch,dera}}$  is computed based on the baseline schedule in day-ahead market using a splines interpolation method as shown in (74).

DERA dispatch is computed at each SCED interval using either profit-following, schedule-following optimization, or self-schedule method. The optimization adopts the target dispatch from SCED and the latest available pricing. Once a dispatch value is determined, it is fixed to this value for future time intervals. There is also an option to limit the look-ahead dispatch window to reduce overall computation time.

### 4.2.1 Schedule-following Method

In the schedule-following method, at each time  $t$ , DERA solves the deviation minimization problem formulated in section 3.3.3. The per-period minimization process starts from  $t = 1$  until the end of the time horizon ( $t = T$ ). During each period  $t$ , the optimization process will seek to adjust dispatch variables (i.e., battery charge/discharge, solar generation, DRRs.) to minimize total deviation of dispatch from the schedules for that specific period, which is formulated as in (70)-(73). Once the dispatch for  $t$  is solved, those values are fixed, and the minimization moves to next period  $t + 1$ . With this rolling-horizon optimization process, the sequential dispatch simulation is generated.

### 4.2.2 Profit-following Method

In the profit-following method, the optimal dispatch of DERA is to maximize its total profit at each time interval, which is formulated as in Section 3.3.1. Because the DERA dispatch ( $p_t$ ) in the profit following is always the same as in baseline schedules from maximizing DERA profit,  $p_t$  is disaggregated into the device-level baseline schedules.

## 5 Future Development

As described in the previous sections, the SCUC-DER simulation platform allows detailed modeling of DER aggregators participating in wholesale electricity markets. Additional updates are planned to broaden this capability and to provide more comprehensive analysis. We summarize some of these planned developments below.

- Integration with regional-scale SCUC datasets
- Mapping of regional datasets to PNNL's taxonomy feeders
- DERA self-schedule offers
- Dynamic distribution factor adjustment for multi-node DERA offers

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