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Integration of wholesale and retail electricity markets: Modeling and stability analysis

April 2017

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U.S. DEPARTMENT OF ENERGY

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Executive Summary

As power systems continue to adapt to the ever-increasing penetration of renewable generation sources, there is a significant increase in the need for reserves to balance the intermittency of renewable generation sources. In the current operational scenario, this would require expensive fast-ramping generators (like gas generation) which may even negate the benefits brought by renewables in terms of carbon emissions. A cheaper and cleaner alternative is to engage flexible loads (electric vehicles, HVACs, water heaters) and distributed energy storage (Tesla powerwall, batteries). Engaging these distributed energy resources in a scalable and manageable fashion will require restructuring of electricity markets. A natural architecture in this scenario is a hierarchy of markets with the highest level corresponding to a wholesale market at the transmission level and the lowest level corresponding to small retail-level markets located on the distribution system.

The hierarchical market structure will involve new types of market interactions and will need a new framework to analyze impacts of various market designs and guide market operators in structuring future electricity markets. Our goal in this project is to build a general framework to analyze the interaction between multiple markets organized in a hierarchy. Markets in each layer of the hierarchy interact with layers above them by bidding aggregate demand/supply curves and receiving as input prices and power dispatch set-points. In this report, we focus on the special case of such a market hierarchy with just two layers: A wholesale transmission level electricity market connected to a number of retail level electricity market and study the interaction between these markets (see figure (1)). The study presented in this report is meant to illustrate possible issues in retail-wholesale integration and lays the foundation for further investigations into the design and analysis of hierarchical market architectures.

The *main contribution* of this report is to develop a theoretical model of the interaction between retail and wholesale markets and to analyze properties of this dynamical model. We provide a brief overview of our model and analysis in the subsequent paragraphs.

We assume that the retail market is responsible for engaging price-responsive loads and bidding the aggregate flexibility into wholesale (transmission level) market. In order to aggregate the flexible loads, the retail markets organizes loads into one of several "flexibility buckets" (see figure (2)). A flexibility bucket represents a collection of loads with similar price elasticity. The retail market constructs an aggregate bid for each of the flexibility buckets and submits the bid to the wholesale market. The wholesale

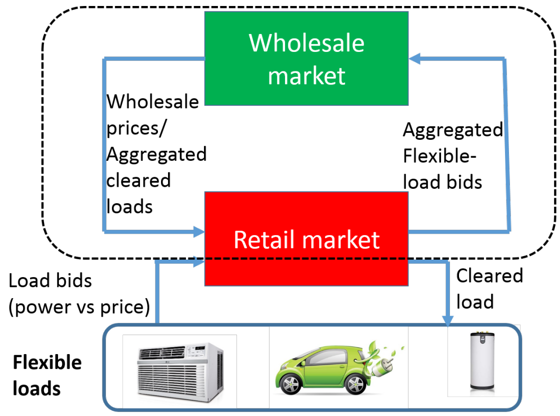

Figure 1: Interactions between wholesale market, retail market and individual devices. The feedback loop inside the dashed black boundary is the object of this report's focus.

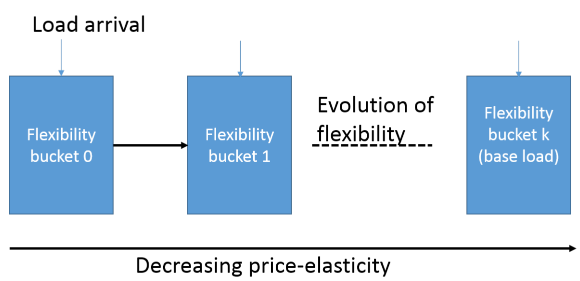

Figure 2: Loads organized into "flexibility buckets" in order of decreasing price elasticity going left to right

market solves an economic dispatch (DCOPF) to compute the cleared load and the prices for each market period. Based on the cleared load value, the retail market operator decides how much each market participant is allowed to consume. The total flexible load that was not cleared for consumption in the current market period is referred to as the "uncleared load".

The flexibility buckets also model the change in price elasticity of loads over time. For example, an electric vehicle would become less elastic as it approaches its charging deadline (since it has to be fully charged by the charging deadline no matter what the price of electricity is). This is modeled by assuming that the uncleared load in market period t moves to the next flexibility bucket (with lower price elasticity) in market period $t+1$ (see

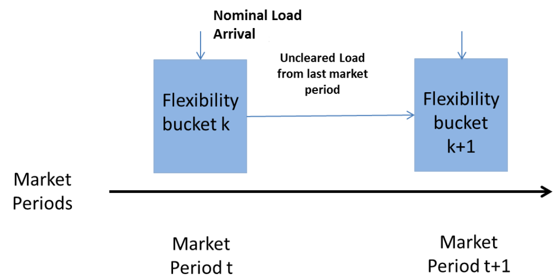

Figure 3: Uncleared flexible load at time t migrates to bucket with lower elasticity at time $t+1$

figure 3). This migratory behavior by the flexible loads, when combined with interaction of the retail market with the wholesale market creates a dynamical system. It is then of interest to understand the behavior of this dynamical system - this analysis will be useful in determining the impact of the market architecture on price volatility, generator dispatch, congestion in the transmission system etc.

In this report we study several natural questions regarding properties of the dynamical system induced by the interaction of the retail and wholesale markets: Under what conditions is the hierarchical market design guaranteed to produce stable prices and generator dispatch levels? Are there situations when prices and dispatch levels can oscillate going from one market period to next (this is naturally an undesirable phenomenon both from a physical and economic perspective)?

We show that the dynamical system converges to a stable set of prices and dispatch levels quickly and cannot produce oscillatory behavior. Furthermore, we provide numerical examples demonstrating the robustenss of these results even when some of the assumptions made in the analysis do not hold. In particular, we study stability of prices when the retail market operator is not exactly able to follow the cleared load dispatch (because of internal dynamics of loads like ACs/HVACs) and when there is congestion in the transmission system. The results show that the predictions made by our analysis (prices converging to stable levels quickly) hold true even when the assumptions of the analysis may be violated and establish that the analysis is likely robust to breaking these assumptions.

The conclusions of this report lay the foundation for further investigations involving more realistic retail market data (that can be used to fit the parameters of the flexibility buckets model) and more complicated hierarchical market architectures that go beyond the simple 2-level architecture studied here.

Contents

1 Background

In this section, we survey reviewing previous work done in retail market design and stability analysis of electricity markets. Retail-side markets effectively aggregate flexible demand and distributed energy resources (DERs) and bid the aggregate flexibility into wholesale markets. Several models of retail markets for engaging flexible loads have been proposed and studied: Proposals under the New York “Reforming the Energy Vision” program can be found in [1] and transactive control mechanisms have been studied in [2][3][4]. Some transactive schemes have even been deployed in pilot demonstration projects [5][6][7].

Several papers in the literature have studied the problem of stability of markets on either the retail or wholesale side. The work presented in [8] demonstrates that direct coupling of consumers to the a retail market without appropriate safeguards in place can result in the highly volatile price behaviour. In [9], the authors extend this work and study a market where flexible consumers predict electricity prices and compute optimal-consumption schedules in response. The authors show that under certain design assumptions, adding memory effects to customer response can enhance stability. A transactive control model that can efficiently coordinate demand-side flexible resources is developed and studied in [10]. Recent work [11] demonstrates that certain implementations of the transactive control model can cause sustained oscillations in demand due to load synchronization. It is shown in [12] that load synchronization tends to occur when controlling deferable loads and algorithms are developed which mitigate volatility and instability issues. One of the key features that distinguishes our work from [8, 11, 12] is the assumption that the bids of the price-responsive loads are taken into account while clearing the wholesale market, rather than devices responding autonomously to price signals. This places the burden of accounting for device level constraints on the retail market operator and reduces a significant potential source of price volatility (viz the devices may not follow the dispatch computed in the wholesaler market). By aggregating several flexible loads on a distribution system, the retail market operator can reduce overall uncertainty and present a simple aggregate model of flexible loads to the wholesale market.

The rest of this report is organized as follows: In section 2, we describe how we model the retail and wholesale electricity markets. In section 3, we present our main theoretical results analyzing stability of the retail-wholesale interconnection. In section 4, we validate our analysis using numerical simulations and show that the conclusions of our theoretical analysis remain valid in general (even when some assumptions made in our theoretical analysis are violated). Finally, in section 5, we summarize our findings and outline directions for future work.

2 Modeling Retail and Wholesale Markets

In this section, we introduce detailed mathematical models of the retail and wholesale markets that form the basis of our study.

2.1 Wholesale market with dispatchable loads

The transmission level wholesale market collects bids from the retail market and the generators and solves the standard economic dispatch problem to determine prices and dispatch levels. A mathematical model of this wholesale market is presented in the subsequent paragraphs.

The set of generators is denoted by $\mathcal{G}$, the set of (aggregated) price-responsive loads by $\mathcal{L}^f$ and the set of inelastic loads by $\mathcal{L}^b$. g_i denotes the power produced by the generator i and must satisfy $g_i \in [G_i^{min}, G_i^{max}]$. Associated with each generator is also a cost function

$$c_i\left(x\right) = \alpha_i^g x^2 + \beta_i^g x \tag{1}$$

where $\alpha^g, \beta^g \geq 0$ are positive parameters. Similarly, the consumption of the i-th flexible load are denoted by l_i^f and must satisfy $l_i^f \in [0, L_i^f]$. Associated with each flexible load is a utility function

$$u_i\left(x\right) = -\alpha_i^f x^2 + \beta_i^f x \tag{2}$$

where $\alpha^f, \beta^f \geq 0$ are flexible load parameters. [1]

Given the cost and utility curves, the market operator solves an economic dispatch problem in order to determine the optimal generation and flexible load levels. The problem is stated as follows:

$$\begin{aligned}
&\min_{l_f, g} \sum_{i \in \mathcal{G}} c_i(g_i) - \sum_{i \in \mathcal{L}^f} u_i(l_i^f) && && \text{(3a)}\\
&s.t \sum_{i \in \mathcal{G}} g_i = \sum_{i \in \mathcal{L}^f} l_i^f + \sum_{i \in \mathcal{L}^b} l_i^b && \text{(Energy balance)} && \text{(3b)}\\
&G_i^{min} \leq g_i \leq G_i^{max} \quad \forall i \in \mathcal{G} && \text{(Generation limits)} && \text{(3c)}\\
&0 \leq l_i^f \leq L_i^f \quad \forall i \in \mathcal{L}^f && \text{(Flexible load limits)} && \text{(3d)}
\end{aligned}$$

[1] The utility function is computed by integrated a linear price-responsiveness curve: $l_i^f = L_i^f - \kappa\lambda$ where λ is the price and L_i^f is the consumption when $\lambda = 0$ (power is free in this case, leading to maximum consumption). This curve can be inverted: $\lambda = \frac{L_i^f - l_i^f}{\kappa}$ to compute the marginal value of consuming the next dl units of power after having consumed l_i^f units of power. Following standard economic theory, the overall utility of consuming x units of power is then $u_i\left(x\right) = \int_0^x \frac{L_i^f - t}{\kappa} dt = \frac{L_i^f}{\kappa} x - \frac{1}{2\kappa} x^2$ so that $\alpha_i^f = \frac{1}{2\kappa}, \beta_i^f = \frac{L_i^f}{\kappa}$.

The optimal solution to this problem as a function of the parameters L^f, l^b is denoted as $l^{f\star}\left(L^f, l^b\right), g^\star\left(L^f, l^b\right)$.

The optimal generation levels and load levels, obtained by solving the problem (3). In most wholesale electricity markets, this problem is augmented with transmission line limits that use the DC power flow approximation and impose limits on the DC approximation of the power flow over transmission lines. In this preliminary work, we ignore the transmission line congestion constraints in our analysis but perform numerical studies showing that accounting for these does not significantly change the conclusions derived from our theoretical analysis of the uncongested case.

The optimization problem (3) is a convex optimization problem with a strongly convex objective function. Hence, it has a unique optimal solution [13]. The dual variable corresponding the constraint (3b) represents the *marginal increase in cost* for satisfying an additional unit of load at the optimal solution. We denote the dual variable as λ and it corresponds to the *market-clearing price.* In the presence of congestion constraints, the price is different at every node of the power system and is known as the *locational marginal price (LMP).* In our theoretical analysis, we will assume a uniform price λ at every bus.

2.2 Retail Market model and dynamics

Retail markets play the role of aggregating the flexibility of a large collection of flexible loads (for example all loads on a distribution network) and bidding the aggregate flexibility into the wholesale electricity market. The key idea we use is that flexibility of loads depends on the physical state of each load - for example, an air conditioner has flexibility (can turn on or off) if the temperature is away from the comfort limits set by a customer. However, as the temperature evolves over time and gets close to one of the comfort limits, the air conditioner is forced to turn on or off and loses flexibility. In the subsequent paragraphs, we develop a simple model to represent large collections of loads with these characteristics.

In order to capture varying levels of price responsiveness among the flexible loads, we assume that the retail markets organize loads into one of several "flexibility buckets" (as discussed in Section ()). Load arrives into any one of several "flexibility buckets" sorted from highly price elastic to completely inelastic.

Formally, a flexibility bucket is a collection of loads with the same (or similar) responsiveness to price that can be modeled using a single price-versus-demand curve: $l_i^f(\lambda) = L_{f;i} - \lambda\kappa_i$ where l_i^f denotes the desired consumption in bucket i at price λ and $L_{f;i}$ is the total load in bucket i. As discussed in the previous section, such a curve can always be turned into a utility function and incorporated into the wholesale market. κ_i represents the "level of price-responsiveness" in bucket i. The buckets are arranged in decreasing order of price-responsiveness, with the first bucket $i = 0$ corresponding to the highest level of price-responsiveness and the final bucket $i = M$ corresponding to the inelastic (base) load with $\kappa_M = 0$.

The evolution of price elasticity of loads is captured in the following dynamical model: Let $L^f i\,(t)$ denote the amount of flexible load in bucket i at time t and $l_i^f\,(t)$ denote the amount of flexible load in bucket i that was *cleared* (satisfied) at time t (computed by the wholesale market). Note that t indexes clearing periods of the wholesale-level electricity market (that is, t advances by 1 every time the wholesale market clears). We denote by $r_i\,(t)$ the amount of *fresh load* arriving into bucket i at time t (for example, a new electric vehicle arriving at a charging station). We then have the following dynamics:

$$L_0^f\,(t) = r_0\,(t) \tag{4a}$$

$$L_i^f\,(t+1) = r_i\,(t) + L_{i-1}^f\,(t) - l_{i-1}^f\,(t)\,, i = 1, \ldots, M \tag{4b}$$

$$l^b\,(t) = L_M^f\,(t) \tag{4c}$$

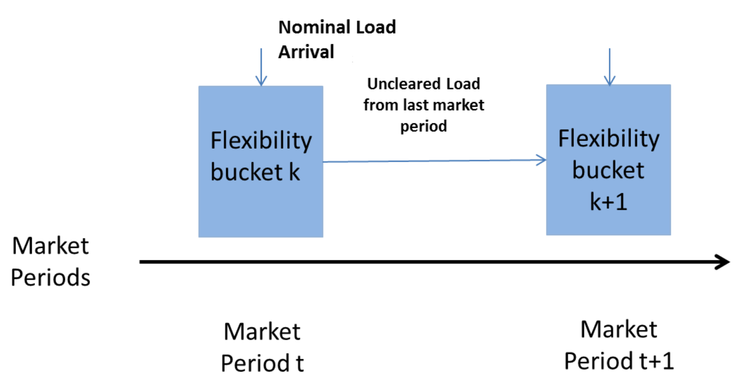

Figure 4: Uncleared flexible load migrates to less flexible bucket over multiple market periods

The first equation says that the amount of flexible load in bucket 0 at time t is equal to the arrival rate into bucket 0 at time t. The second equation says that the the amount of load in bucket $i = 1, \ldots, M$ at time

$t+1$ is equal to the arrival rate at time $t+1$ plus the uncleared load in bucket $i-1$ at time t (since the uncleared load in bucket $i-1$ from the previous time-step moves into bucket i) . The final equation says that load in the final bucket M defines the value of the base load l^b in the retail market. Therefore, in each market cycle, a fraction of the flexible load in each bucket is cleared (along will all of the base load). The remaining (uncleared) flexible load moves into a subsequent bucket (with lower price elasticity) in the next market period (See figure 4).

The dynamics (4) coupled with the wholesale market clearing procedure (3) (run at each time-step t with the given values of L^f, l^b) defines a dynamical system describing the wholesale-retail interconnection. At every time step t, flexible load values $L^f(t)$ (arising from the dynamics (4)) are submitted by the retail market to the wholesale market which then solves the economic dispatch (3) to computed the optimal cleared load values $l_i^f(t)$ in each bucket and the optimal generation dispatch $g_i^\star(t)$, along with the market clearing price $\lambda(t)$.

Finally, we note that while the model present here assumes a single retail market connected to a wholesale market, our analysis and conclusions extend in a straightforward manner to a setup with *several retail markets connected to a wholesale market* (perhaps with different buses in the wholesale market corresponding to distinct retail markets).

3 Stability Analysis of Retail/Wholesale interaction

In this section, we provide a stability analysis of the market dynamics defined by (3) and (4) under certain simplifying assumptions. The *state* of this dynamical system is the values of the load in each bucket at any given time and the prices and generation dispatch levels at any given time are functions of the state. In this report, stability of a dynamical system refers to a control-theoretic characterization of the long-term behavior of the dynamical system and answers questions such as : Do the states grow without bound? Do the states exhibit oscillatory behavior? The results presented in the subsequent sections show that the market dynamics developed in the previous section converges to some equilibrium and does not exhibit oscillatory behavior (we prove this theoretically under a set of simplifying assumptions and numerically in more general cases).

3.1 Main Theoretical Results

In this subsection, we present results that analyze the stability of the dynamical system defined by (3) and (4). We study the special case where the arrival rate $r_i(t)$ is constant over time - this is critical for analyzing a stationary dynamical system (otherwise the equilibrium of the system is always changing and stability does not have a well-defined meaning). Practically, this can be justified as long as the market dynamics converges on a much faster timescale than the rate at which new loads arrive and load flexibility evolves (we provide some numerical examples demonstration this in section 4). The overall dynamics is given as follows:

$$L_0^f(t) = r_0 \tag{5a}$$
$$l_i^f(t) = l_i^{f\star}\left(L^f(t), l^b(t)\right) \quad \forall i = 0, \ldots, M \tag{5b}$$
$$L_i^f(t+1) = r_i + L_{i-1}^f(t) - l_{i-1}^f(t) \quad \forall i = 1, \ldots, M \tag{5c}$$
$$l^b(t) = L_M^f(t) \tag{5d}$$

where $l^{f\star}$ refers to the the optimal solution of the economic dispatch problem (3) given values of L^f and l^b. We can also define relevant outputs of this dynamical system $\lambda(t) = \lambda^\star\left(L^f(t), l^b(t)\right), g(t) = g^\star\left(L^f(t), l^b(t)\right)$. Our main result shows that the dynamics (5) converges to an equilibrium point when initialized at an arbitrary initial condition:

Theorem 1. *For any given values of* r*, the variables* $L_i^f(t)$ *and* $l^b(t)$ *evolving according to the dynamical system* (5) *are non-decreasing and converge to equilibrium values.*

Note that (5) may not have a unique equilibrium point. However, we show that initialized at an arbitrary condition, the dynamics must converge to an equilibrium point and no oscillations are possible (because of the monotone behavior of the system state).

In order to establish this result, we use the monotonicity property of the economic dispatch problem (3) established by lemma 2. The lemma shows that the amount of *uncleared load* $L^f - l^{f\star}\left(L^f, l^b\right)$ is non-decreasing in L^f, l^b. Given this, it is easy to prove the main theorem 1. The proof can be found in the appendix section 6.

Theorem 1 implies that the amount of flexible load (and inelastic load) in each of the "flexibility buckets" increases monotonically and will eventually stabilize to some equilibrium value (since there is a bound on the maximum load in the system). Thus, the system cannot exhibit oscillatory behavior or price volatility (See figure (5) for typical behavior).

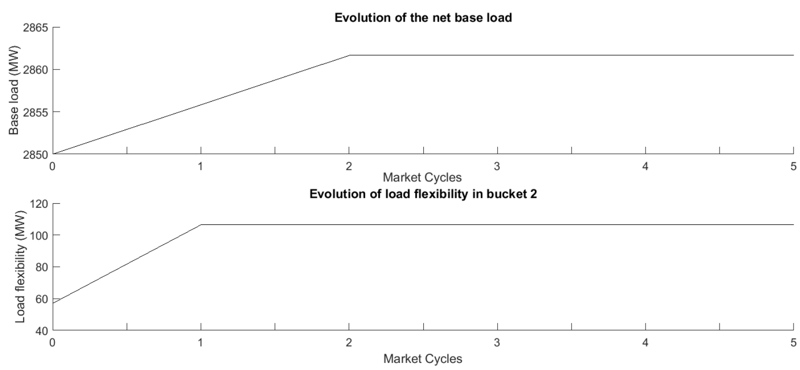

Figure 5: Asymptotic behavior of the retail market dynamics for IEEE RTS-24 system with two flexibility buckets

3.2 Design Considerations

Theorem 1 establishes, that under the absence of transmission line constraints, that the amount of flexible load in the system does not exhbhit oscillatory behavior. From a market design perspective, this guarantees that price fluctuations will not occur due to the presence of flexible load when there is no congestion under the market design that gave rise to the dynamics (5). The effect of adding transmission constraints to the economic dispatch model on the stability of the retail-wholesale dynamics is a subject of ongoing investigation. There are also other choices pertaining to the design of retail market dynamics (5) which is of significant interest to a market designer:

1 *Rate of convergence*: Theorem 1 establishes that the market dynamics (5) converges to equilibrium, but does not provide a convergence rate. The stability analysis assumes that the the arrival rate of inflexible load is constant over time. In reality, the inflexible load varies slowly across market cycles depending on the time of the day. It is therefore important to ensure that the retail market is designed in such a way that the market dynamics (5) converges quickly so that the slowly changing inflexible load does not affect the stability of the system.

2 *Robustness to noise*: Theorem 1 assumes that the retail market model we have is perfect and that the flexible loads on the retail market side precisely follow the dispatch computed by the wholesale market. Since these assumptions may be violated in a real system, it is important to study how robust the convergence is to adding noise to the response of the retail market (so that it follows the wholesale dispatch not perfectly, but with some noise).

In the following subsection, we present simulations that investigate the above two issues not covered by our theoretical results that are relevant to the stability of the retail-wholesale interconnection.

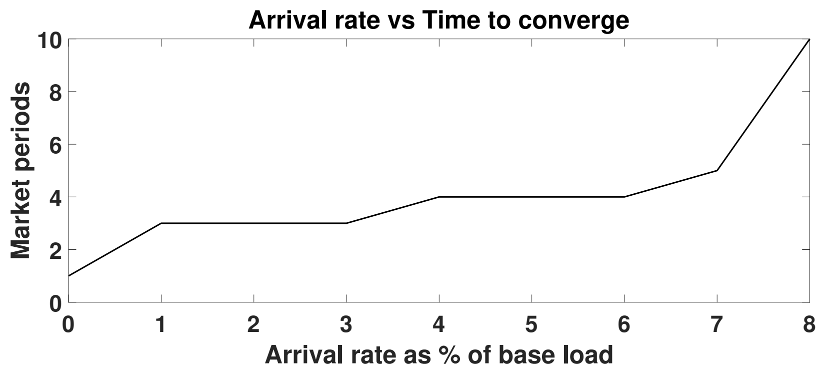

Figure 6: Number of market periods before convergence of base load is observed

4 Numerical Studies

In this subsection, we present numerical simulations studying the properties of the dynamical system defined by the retail and wholesale markets on a IEEE RTS-24 system (via MATPOWER [14]).

4.0.1 Rate of convergence

The theorems presented in the previous section establishes the stability of the dynamical system (5). The numerical study presented here will provide some insight into the rate of convergence of the system.

The base load l^b for the IEEE 24-bus system is set to the nominal power demand value in the case file. We then run the dynamics (5) and record how long the dynamics takes to converge, as a function of the percentage of flexible load in the system (which is quantified in terms of the arrival rates $r_i(0 \leq i \leq M-1)$ into the flexible buckets as a percentage of the base load rate r_M). Fig (6) shows the number of market periods required for the system (5) to converge when the arrival rate is increased (expressed a percentage of the base load in (6)). It can be seen that as the arrival rate increases, the convergence rate slows down. This can be explained using lemma 2 - as the arrival rate increases l^b, L^f increase (since these quantities are monotonic in the arrival rate) and hence by lemma 2 the *uncleared load increase* so that more uncleared load remains in each market period. Hence, it makes sense that it takes longer for the system to converge to equilibrium. Exact characterization of the convergence rate in terms of the model parameters is a direction of future work that we will pursue.

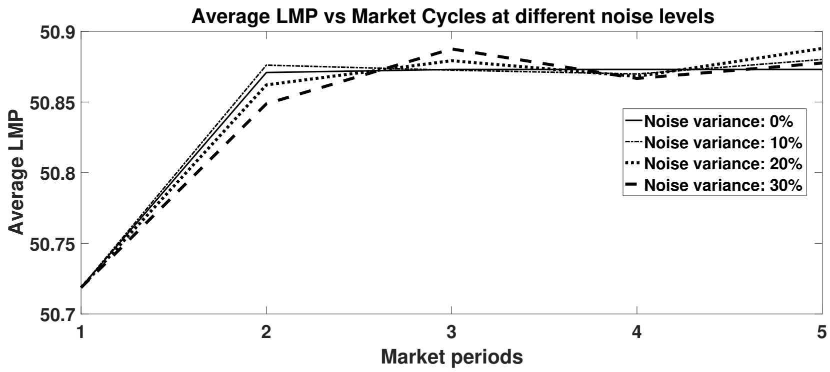

Figure 7: Evolution of the mean of LMP over market periods at different noise levels

These results also show that the assumption of constant arrival rates is reasonable since unless the arrival rate is a significant fraction of the base load, the dynamics converge to equilibrium within a few market periods. Since we study real-time markets (5-15 minutes), one would not expect significant changes in arrival rates on these timescales.

4.0.2 Robustness to noise

Theorem 1 establishes the stability of (4) assumes that the flexible load dispatch $l^{f\star}$ is followed perfectly. However, given that flexible loads have their own internal dynamics and control logic, it is likely that the retail market operator will not be able to schedule loads to perfectly follow the dispatch. Hence, we have

$$l_i^f\left(t\right) = l_i^{f\star}\left(L^f\left(t\right), l^b\left(t\right)\right) + \omega_i(t)$$

where ω denotes noise. We model ω as zero-mean Gaussian noise and examine how this noise propagates through the dynamic 5. The variance of the noise $\omega_i(t)$ added at bucket i is equal to a percentage of the default arrival rate r_i at that bucket and r_i is chosen as to be 5% of the base load arrival rate r_M.

Figure (7) shows the impact of the noise introduced on the prices. In figure 7, we plot the price trajectory (averaged over several realizations of the noise) as a function of the noise variance - the results show that price trajectories remain close to the price trajectory with $\omega = 0$. Furthermore the time-averaged variance of the prices is a slowly increasing function of

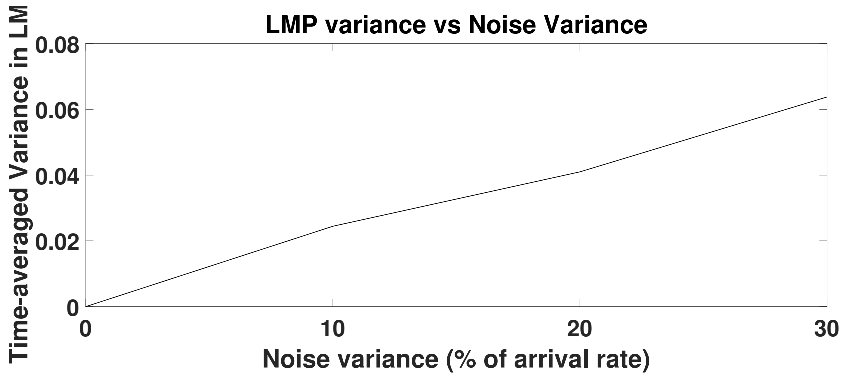

Figure 8: The time-averaged variance of LMPs at different noise levels

the variance of the noise (as seen in 8) - this shows that the system is likely robust to noise and does not amplify 0-mean noise to create instabilities. Theoretical investigation of this phenomenon is also a direction of future work.

5 Conclusions and Future work

We have developed a general abstract model of retail markets and used it to study the interface between wholesale and retail electricity markets. Our theoretical results show that under our model of the retail market dynamics, there are no oscillations between the wholesale and retail markets assuming absence of congestion and constant load arrival rates. Our numerical simulations indicate that these conclusions are valid even without the assumptions made. Extending our theoretical framework to explain the observations in section 4 is a natural direction of future work.

We envision that the contributions made in this report have several applications: The model of retail markets developed here can serve as a useful starting point for more detailed investigations. One interesting direction of research would be collect data from an actual retail market (or a detailed agent-based simulation of a retail market) and fit parameters of the flexibility buckets model developed in this report. This would enable us to characterize how well this model captures the dynamics of flexible loads in real markets and analyze how deviations from this model impact stability (since the simulations in this report seem to indicate that stability holds even under mild perturbations from the flexibility buckets model). We envision that this work will lay the foundation for a comprehensive framework for the design and analysis of hierarchical market designs and will prove useful to market operators and policy makers working on electricity market reform for future smart grids with significant penetration of renewable generation sources and flexible demand-side resources.

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6 Appendix

Lemma 1. *The optimal solution* $l_i^{f\star}\left(L^f, l^b\right)$ *to (3) is:*
a) a continuous function of l^b *and* L^f*.*
b) a decreasing linear function of the parameters l^b *and* L^f *if* i *is* marginal*, ie,* $0 < l_i^{f\star}\left(L^f, l^b\right) < L_i^f$*.*

Proof. The continuity of the optimal solution $l_{f,i}^*$ follows directly from Berge's maximum theorem.

The decreasing nature of the $l_i^{f\star}$ when marginal is established as follows:

Let

$$N_g^+(l_b, L_{f,i}) = \{j \mid j \in \mathcal{N}, g_j^* = G_j^{max}\} \tag{6}$$

$$N_g^-(l_b, L_{f,i}) = \{j \mid j \in \mathcal{N}, g_j^* = G_j^{min}\} \tag{7}$$

$$N_l^+(l_b, L_{f,i}) = \{j \mid j \in \mathcal{N}, l_i^{f\star} = L_i^f\} \tag{8}$$

$$N_l^-(l_b, L_{f,i}) = \{j \mid j \in \mathcal{N}, l_i^{f\star} = 0\} \tag{9}$$

represents the set of all non-marginal generators and loads when the base load is set to $l_b(t)$ and the bounds on the dispatchable load is set to $L_{f,i}$. The set of marginal generators is then given by $\mathcal{M}_g = \mathcal{N} - (N_g^+ \cup N_g^-)$ and the set of marginal loads is given by $\mathcal{M}_l = (\mathcal{L}_f - (N_l^+ \cup N_l^-)$. (Recall that $\mathcal{L}_f$ denotes the set of elastic loads.)

Let $m \in L_f$. Now suppose, $m \in \mathcal{M}_l$ (i.e $0 < l_i^{f\star}\left(L^f, l^b\right) < L_i^f$). It can be shown that $l_i^{f\star}$ is a decreasing function of the base load l_b and the bounds on the flexible load $L_{f,i}$.

The KKT conditions for any marginal generator z in $\mathcal{M}_g$ is as follows:

$$\frac{\partial L}{\partial g_z} = 2\alpha_z^g g_z^* + \beta_z^g + \lambda = 0 \tag{10}$$

where λ is dual variable corresponding to the power balance constraint. Furthermore, the dual variables corresponding to the inequality constraints are zero as $z \in \mathcal{M}_g$ implies that the corresponding generator is marginal. Similarly, writing down the KKT conditions for any $y \in \mathcal{M}_l$, we obtain

$$\frac{\partial L}{\partial l_y^f} = 2\alpha_y^f l_y^{f\star} - \beta_y^f - \lambda = 0 \tag{11}$$

This implies that for any $z \in \mathcal{M}_g$, we have

$$2\alpha_m^f l_m^{f\star} - \beta_m^f = \lambda = -2\alpha_z^g g_z^* - \beta_z^g \tag{12}$$

$$\implies g_z^* = \frac{2\alpha_m^f l_m^{f\star} - \beta_m^f + \beta_z^g}{-2\alpha_z^g} \tag{13}$$

$$\implies g_z^* = -\alpha_{m,z} l_{f,m}^* + c_{m,z} \tag{14}$$

where $\alpha_{m,z}, c_{m,z} > 0$.

Similarly, for any $y \in \mathcal{M}_l$, we have

$$2\alpha_m^f l_m^{f\star} - \beta_m^f = \lambda = 2\alpha_y^f l_y^{f\star} - \beta_y^f \tag{15}$$

$$\implies l_y^{f\star} = \frac{2\alpha_m^f l_m^{f\star} - \beta_m^f + \beta_y^f}{2\alpha_y^f} \tag{16}$$

$$\implies l_y^{f\star} = \gamma_{m,y} l_m^{f\star} + d_{m,y} \tag{17}$$

where $\gamma_{m,y} > 0$.

Substituting for g_z^* and $l_y^{f\star}$ and accounting for the non-marginal loads and generation, we obtain

$$\sum_{z \in N_g^+} G_z^{max} + \sum_{z \in N_g^-} G_z^{min} + \sum_{z \in \mathcal{M}_l} -\alpha_{m,z} l_m^{f\star} + c_{m,z} =$$
$$\sum_{i=1}^{n} l_{b,i} + \sum_{y \in N_l^+} L_{f,y} + \sum_{y \in M_l} \gamma_{m,y} l_m^{f\star} + d_{m,y}$$

Rearranging, we obtain

$$l_m^{f\star} = \frac{l_b + \sum_{y \in N_l^+} L_{f,y} + C}{\sum_{z \in \mathcal{M}_l} -\alpha_{m,z} - \sum_{y \in M_l} \gamma_{m,y}} \tag{18a}$$

$$C = \sum_{y \in M_l} d_{m,y} - \sum_{z \in N_g^+} G_z^{max} - \sum_{z \in N_g^-} G_z^{min} - \sum_{z \in \mathcal{M}_l} c_{m,n,z} \tag{18b}$$

The slope $\dfrac{1}{\sum_{z \in \mathcal{M}_l} -\alpha_{m,n,z} - \sum_{(y,z) \in M_l} \gamma_{m,n,y,z}}$ in (18) is negative and implies that a net increase in the base load results or an increase in the capacity of the non-marginal dispatchable loads results in a decrease in the marginal dispatched load which was to be shown. □

Lemma 2. *Let L^f and $\tilde{L}^f$ be such that $L^f \leq \tilde{L}^f$ component-wise and let $l^b \leq \tilde{l}^b$. Then, we have*

$$L_i^f - l_i^{f\star}\left(L^f, l^b\right) \leq \tilde{L}_i^f - l_i^{f\star}\left(\tilde{L}^f, \tilde{l}^b\right)$$

Proof. Define $L_i^f(s) = sL_i^f + (1-s)\tilde{L}_i^f$ and $l^b(s) = sl_i^b + (1-s)\tilde{l}_i^b$ and $s \in [0,1]$. Let $h(s) = L^f(s) - l^{f\star}\left(L^f(s), l^b(s)\right)$ denote the uncleared load as a function of s.

We show that $g_i(s)$ is non-decreasing. Suppose, there exists (s_1, s_2) such that $g_i(s)$ is decreasing for all $s \in (s_1, s_2)$. This implies that flexible load l_i^f is marginal and increasing in (s_1, s_2) which contradicts Lemma (1). As such, the amount of uncleared load $g_i(s)$ is a non-decreasing function of s. Then, $g_i(0) \leq g_i(1)$ - hence the theorem. □

6.1 Proof of theorem 1

Proof. The proof proceeds by showing that the base load $l^b(t)$ and $L^f(t)$ are increasing and bounded above which guarantees that they will converge. This is done via induction.
Assume $l^b(t-1) \leq l^b(t)$ and $L^f(t-1) \leq L^f(t)$. Then, we claim $l^b(t) \leq l^b(t+1)$ and $L^f(t) \leq L^f(t+1)$. By Lemma (2):

$$L^f(t-1) - l^f(t-1) \leq L^f(t) - l^f(t) \tag{19a}$$

$$\implies L^f(t) \leq L^f(t+1) \tag{19b}$$

where the second implication follows since $L^f(t+1)$ depends monotonically on $L^f(t) - l^f(t)$. A similar argument can be used to establish the monotonicty of $l^b(t)$. Now, if it can be shown that $l^b(0) \leq l^b(1)$ and $L^f(0) \leq L^f(1)$, the proof is complete. Since $l^b(1) = r_M + c \geq l^b(0)$ where c is a postive constant, it follows that $l^b(0) \leq l^b(1)$. Similarly, $L_i^f(1) = r_i + d \geq L_i^f(0)$ where d is a positive constant. This completes the base case of the induction and establishes monotonicity of the $l^b(t), L^f(t)$. Furthermore, $L_i^f(t) \leq \sum_{k=0}^{i-1} r_i$ and $l^b(t) \leq \sum_{i=0}^{M} r_i$. As such, $l^b(t)$ and $L^f(t)$ are increasing sequences bounded above and hence must converge. □

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